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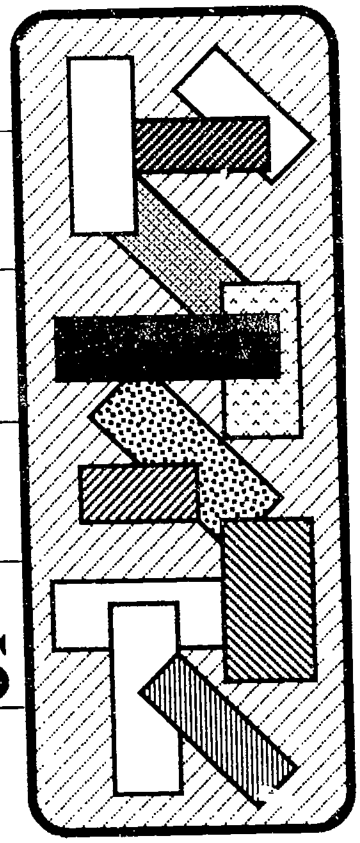
ABSTRACT

This document is designed to assist teachers and other school personnel in the planning and teaching of the third grade mathematics course. Contents include: (1) Overview of Grade 3 Mathematics (mission statement, purpose and philosophy, goals, National Council of Teachers of Mathematics' Professional Standards for Teaching Mathematics, instructional strategies, and uses of technology and manipulatives); (2) Essential Elements of Instruction with sample learning objectives and sample clarifying activities; (3) Texas Assessment of Academic Skills (TAAS) (focus, domains, objectives, and targets); (4) Sample Lessons for Teaching Grade 3 Mathematics; and (5) Evaluation (philosophy and types of evaluation). TAAS features three domains: concepts, operations, and problem solving. The Essential Elements are: problem solving; patterns, relations, and functions; number and numeration concepts; operations and computation; measurement; geometry; and probability, statistics, and graphing. Suggested resources include children's trade books, software, and suggested manipulatives. Contains 22 references. (MKR)

GUIDELINES FOR TEACHING GRADE 3 MATHEMATICS



Texas Education Agency
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Fall 1994

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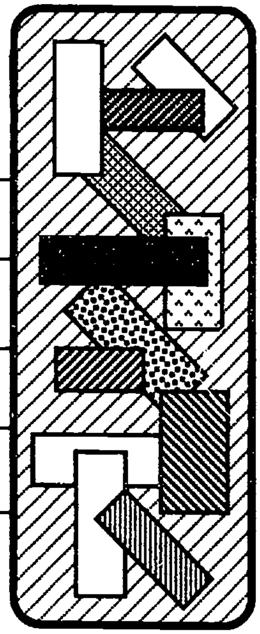

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FOREWORD

Guidelines for Teaching Grade 3 Mathematics is designed to help teachers and other school district personnel plan and teach third grade mathematics. The publication presents the philosophy and intent of the course and discusses the required essential elements, TAAS instructional targets, instructional strategies, and the use of technology and manipulatives. Also included are sample objectives and activities to illustrate how the essential elements for third grade mathematics can be taught. School district personnel may want to use these suggestions to develop their own curriculum documents for the course.

We hope these guidelines will be useful in planning and teaching mathematics in Grade 3 and in equipping the mathematics classroom.

Lionel R. Meno
Commissioner of Education

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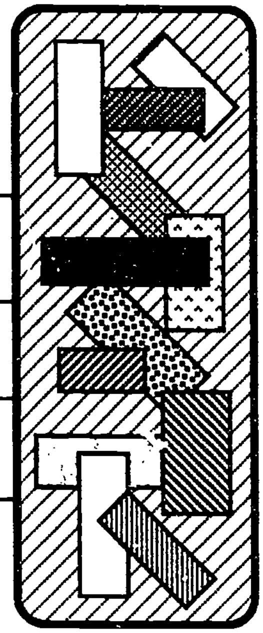
Overview of Grade 3 Mathematics



Mission Statement

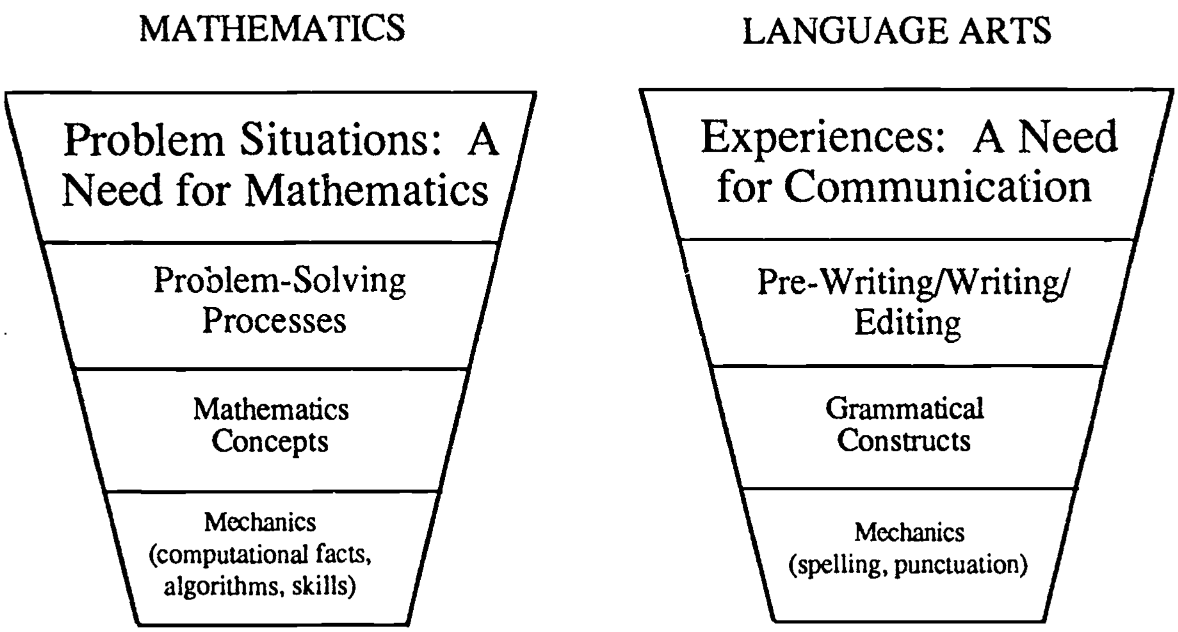
Guidelines for Teaching Grade 3 Mathematics is one in a series of eight documents for the first through the eighth grades designed to assist teachers and other school personnel in the planning and teaching of elementary mathematics. The discussions of philosophy, goals, instructional strategies, uses of technology and manipulatives, and aspects of evaluation are provided as starting points for districts to begin the process of developing their own curriculum documents. The essential elements of instruction for each grade level are supported with sample learning objectives, sample clarifying activities, and complete sample lessons. These guidelines should prove useful to district personnel in: (1) planning curriculum, (2) planning instruction, and (3) equipping classrooms for mathematics teaching and learning.

Purpose and Philosophy

Mathematics is useful, exciting, and creative and can be enjoyed by all elementary school students. Problem-solving skills and logical reasoning are developed while students explore and make sense of their world through rich, worthwhile mathematical experiences. Unfortunately, mathematics has been viewed by many students as boring, irrelevant, and routine and as externally dictated by a rigid system of rules governed by standards of speed, accuracy, and memory. In the past, computational facility has been emphasized instead of a broad, integrated view of mathematics. While computational skills are important, learner characteristics and the vitality of mathematics itself cannot be overlooked. Mathematics in the elementary grades should be broad-based and concept driven and should reflect relevant mathematics and connections between mathematics concepts and between these concepts and other disciplines.

Children enter elementary school with a natural curiosity and enthusiasm for learning. Mathematics experiences at the elementary level should tap into these characteristics for children to begin developing mathematical power—the ability to think and communicate, drawing on mathematical ideas and using mathematical tools and techniques. The attitudes students form in elementary school toward mathematics will determine the choices they make of future mathematics coursework and consequently the availability or loss of educational and career opportunities.

The elementary school mathematics curriculum should emphasize the processes of problem solving, reasoning, communication, and making connections within the contexts of investigating geometry, measurement, probability, statistics, graphing, patterns, and functions, as well as with number, numeration, and operation concepts. Problem solving should be the focus of instruction with skills and concepts being introduced, developed, and applied through meaningful problem situations. Mathematics instruction needs to begin with meaning and purpose in much the same way as elementary teachers present language arts instruction, as reflected in the following graphic illustration:



All students need rich and relevant problem-solving experiences with appropriate teacher guidance and questioning. Such experiences will empower students to build meaning for the mathematics they encounter today and to strengthen reasoning skills needed for the mathematics of tomorrow.

Goals

According to *Curriculum and Evaluation Standards for School Mathematics* developed by the National Council fo Teachers of Mathematics (NCTM), the five overall curriculum goals for students are:

- learning to value mathematics
- becoming confident in their ability
- becoming mathematical problem solvers
- learning to communicate mathematically

- learning to reason mathematically

Moreover, the educational system of today demands new societal goals for education:

- mathematically literate workers
- lifelong learning
- opportunity for all
- an informed electorate

Specifically, teaching the mathematics curriculum to elementary school students must be related to the characteristics of the learners and their needs today and in the future.

Everybody Counts (National Research Council, 1989) posits that "self-confidence built on success is the most important objective of the mathematics curriculum" (p. 45). Individuals must be able to use mathematics in their later lives—as employees, parents, and citizens. Ability and disposition to do so often depends on attitudes toward mathematics developed in school.

Through the use of worthwhile mathematical activities investigated in cooperative, group environments, teachers of elementary mathematics can empower their students with strong mathematical understanding and disposition.

National Council of Teachers of Mathematics: Professional Standards for Teaching Mathematics

The *Professional Standards for Teaching Mathematics (NCTM, 1991)* are based on four assumptions about the practice of teaching. These assumptions are abbreviated versions of the more extensive ones found in the original document (NCTM, 1991, pages 21-22).

(1) The goal of teaching mathematics is to help all students develop mathematical power. Teachers must help every student develop conceptual and procedural understandings of number, operations, geometry, measurement, statistics, probability, functions, and algebra and the connections among ideas. They must engage all students in formulating and solving a wide variety of problems, making conjectures and constructing arguments, validating solutions, and evaluating the reasonableness of mathematical claims.

(2) What students learn is fundamentally connected with how they learn it. Students' opportunities to learn mathematics are a function of the setting and the kinds of tasks and discourse in which they participate.

(3) All students can learn to think mathematically. The goals such as learning to make conjectures, to argue about mathematics using mathematical evidence, to formulate and solve problems, and to make sense of mathematical ideas are not just for some group thought to be "bright" or "mathematically able."

(4) Teaching is a complex practice and hence not reducible to recipes or prescriptions. First of all, teaching mathematics draws on knowledge from several domains: knowledge of mathematics, of diverse learners, of how students learn mathematics, of the contexts of the classroom, school, and society. Good teaching depends on a host of considerations and understandings. Good teaching demands that teachers reason about pedagogy in professionally defensible ways within particular contexts of their own work.

The *Professional Standards for Teaching Mathematics* identifies a particular set of instructional standards for the effective teaching of mathematics. The standards describe the nature of the tasks, patterns of communication, the learning environment, and the analysis of instruction. More specifically, five of these standards focus on instructional strategies. They are:

STANDARD 1: WORTHWHILE MATHEMATICAL TASKS

The teacher of mathematics should pose tasks that are based on:

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences;
- knowledge of the range of ways that diverse students learn mathematics;

and that

- engage students' interests;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;

- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics.

STANDARD 2: THE TEACHER'S ROLE IN DISCOURSE

The teacher of mathematics should orchestrate discourse by:

- posing questions and tasks that elicit, engage, and challenge each student's thinking ability;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

STANDARD 3: STUDENTS' ROLE IN DISCOURSE

The teacher of mathematics should promote classroom discourse in which students:

- listen to, respond to, and question the teacher and one another;
- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counterexamples to investigate a conjecture;
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
- rely on mathematical evidence and argument to determine validity.

STANDARD 4: TOOLS FOR ENHANCING DISCOURSE

The teacher of mathematics in order to enhance discourse, should encourage and accept the use of:

- computers, calculators, and other technology;
- concrete materials used as models;
- pictures, diagrams, tables, and graphs;
- invented and conventional terms and symbols;
- metaphors, analogies, and stories;
- written hypotheses, explanations, and arguments;
- oral presentations and dramatizations.

STANDARD 5: LEARNING ENVIRONMENT

The teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by:

- providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems;

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- using the physical space and materials in ways that facilitate students' learning of mathematics;
- providing a context that encourages the development of mathematical skill and proficiency;
- respecting and valuing students' ideas, ways of thinking, and mathematical dispositions;

and by consistently expecting and encouraging students to:

- work independently or collaboratively to make sense of mathematics;
- take intellectual risks by raising questions and formulating conjectures;
- display a sense of mathematical competence by validating and supporting ideas with mathematical argument.

STANDARD 6: ANALYSIS OF TEACHING AND LEARNING

The teacher of mathematics should engage in ongoing analysis of teaching and learning by:

- observing, listening to, and gathering other information about students to assess what they are learning;
- examining effects of the tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions;

in order to:

- ensure that every student is learning sound and significant mathematics and is developing a positive disposition toward mathematics;
- challenge and extend students' ideas;
- adapt or change activities while teaching;
- make plans both short- and long-range;
- describe and comment on each student's learning to parents and administrators, as well as to the students themselves.

The movement toward this vision of instruction for mathematical empowerment of all students is strongly dependent upon the environment of the classroom, an environment governed in a large part by the decision-making role of the classroom teacher. The NCTM teaching standards identify five major components necessary in the instructional environment for the mathematics classroom and tie these components directly to teachers asking, and encouraging students to ask, appropriate and stimulating questions. The five major instructional components and suggestions for questions are (NCTM, 1991, pp. 3-4):

- **Helping students work together to make sense of mathematics**
  "What do others think about what Janine said?"
  "Do you agree? Disagree?"
  "Does anyone have the same answer but a different way to explain it?"
  "Would you ask the rest of the class that question?"
  "Do you understand what they are saying?"
  "Can you convince the rest of us that that makes sense?"

- **Helping students to rely more on themselves to determine whether something is mathematically correct**
  "Why do you think that?"
  "Why is that true?"

"How did you reach that conclusion?"
"Does that make sense?"
"Can you make a model to show that?"

- **Helping students learn to reason mathematically**
  "Does that always work?"
  "Is that true for all cases?"
  "Can you think of a counterexample?"
  "How could you prove that?"
  "What assumptions are you making?"

- **Helping students learn to conjecture, invent, and solve problems**
  "What would happen if . . .." What if not?"
  "Do you see a pattern?"
  "What are some possibilities here?"
  "Can you predict the next one? What about the last one?"
  "How did you think about the problem?
  "What decision do you think he should make?"
  "What is alike and what is different about your method of solution and hers?"

- **Helping students to connect mathematics, its ideas, and its applications**
  "How does this relate to . . .?"
  "What ideas that we have learned before were useful in solving this problem?"
  "Have we ever solved a problem like this one before?"
  "What uses of mathematics did you find in the newspaper last night?"
  "Can you give me an example of . . .?"

Instructional Strategies

The following diagrams are examples of one teacher's planning efforts to connect measurement and geometry concepts to life, earth, and physical science units:

Measurement

Weather
Measure, record
- temperature °C °F
- volume of rainfall
- feet of snow

Dental Health
- Measure time spent brushing teeth
- Measure time spent in dentist office

Human Body
Estimate and measure body parts in inches. Draw a picture of yourself to show results. Estimate the weight of classmates. Organize/sequence from least to most weight. Weigh on scales, collect data, check predictions.

Food Groups
Use foods to help measure capacity of various containers. Weigh and compare fresh fruit vs. dried fruit.

Simple Machines
Use small cars (wheels) and ramps (inclined planes). Race cars off ramps, measure distance in inches, feet. Does incline affect distance?

Ecology
Buy an acre of rainforest. Measure and mark off an acre of land.

Plants
Students keep a record of plant growth, using inches, cm. Meaure weight of various seeds. Record volume of water used to feed plants.

Insects
Students measure wings or length of various insects. Measure length of ant food trails—distance to mound.

Dinosaurs
Use feet to measure length of dinosaurs. Compare capacity; e.g., 1 dinosaure = 1 bus. How do their masses compare?

Animals
Bring in stuffed animals to create a classroom zoo. Sequence animals by height, weight, longest tail, arms, legs, wings, etc. Use string and inches to verify and graph results.

Earth science
Predict the weight sequence of a set of rocks. Use scales to verify the sequence. Weigh different soil types. Do a cup of sand and a cup of top soil weigh the same?

Geometry

Weather
Critical Thinking—Write a paragraph. What if . . . hail were shaped like cones, cubes, pyramids, etc. Create unique cloud shapes using precut construction paper shapes.

Dental Health
Discuss various teeth and their jobs. what shape (3-D) do these teeth resemble? Critical thinking: What if our teeth were all shaped like cones, spheres? Etc.

Animals
Use measurement info on animals to graph results. Use pattern blocks to create unique animals.

Space
Use three-dimensional shapes to make a rocket ship. Use construction paper to create a space-shape monster. Write data identifying the monster's various shape/body parts. Identify planets as spheres.

Human Body
Bilateral symmetry—Art project. Draw one side of body; fold, trace other side. Plot measurements of student height or weight on line graphs.

Simple Machines
Explore angles of inclined planes for racing activities. Does incline affect the distance the car travels? Rotational symmetry of various wheels and gears create efficiency of machines. Use toy wheels, gears, etc. to create a unique machine. What job does it perform?

Insects
Identify symmetry in a variety of insects. Use art to help with this concept; i.e., making butterflies with paint on one side of paper, folding to create mirror image. Look at rotational symmetry in bee hives and at the shape of cells in hives.

Food Groups
Bring in a variety of foods. Have students identify foods that closely match geometric solids: cube, cylinder, cone, pyramid, and sphere.

Plants
Identify the shapes found in real plants; e.g., leaves, petals, stems. Use pattern blocks to create a unique plant. Identify shapes. Plot growth of plants to create a line graph.

Appropriate questioning techniques and meaningful problem-solving situations are two major strategies for effective mathematics instruction.

Uses of Technology and Manipulatives

Calculators and computers are tapped for important roles in mathematics at all levels and across topics. Changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself. The new technology has altered the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

The NCTM *Curriculum Standards* (1989) call for the following regarding technology in the classroom:

- appropriate calculators for all students at all times
- a computer for every classroom for demonstration
- access to a computer for individual and group work
- students learning to use the computer as a tool for processing information and performing calculations to solve problems

Calculators and computers offer teachers and students an important learning aid. Their potential is great and as yet untapped both in developing concepts and in developing positive attitudes and persistence in problem solving.

Computers can be utilized in a variety of ways in the mathematics classroom, and the appropriateness of a particular approach depends on the goals. Three qualitatively different methods suggested by R. Taylor in *The Computer in the School: Tutor, Tool, Tutee* are:

- as a sophisticated teaching machine
- to be programmed (or taught) by the student
- as a mode for applications in research and development through software that displays graphs, manipulates symbols, analyzes data, and performs mathematical procedures. Applications such as spreadsheets, word processing, data bases, and communication packages have the appeal of matching the classroom's use of technology with that of society's.

Calculator use is not for the purpose of replacing paper-and-pencil computations but to reinforce them. According to N. Kober in *Ed Talk: What We Know About Mathematics Teaching and Learning*, calculator use is apt to sustain independent thought, not replace it. For example, students can be challenged to invent calculator algorithms to replace procedures taught in textbooks. The students explain why their procedures work and debate the advantages and disadvantages of their procedures over others. Calculators are programmable, produce graphics, and work in fractional and algebraic notation. Teachers need to be innovative; they need to experiment and share ideas.

Furthermore, manipulatives offer an excellent way to enable students to connect between mathematical ideas. Learning is enhanced when students are exposed to a concept in a variety of manipulative contexts. As an example, fractions represented with pattern blocks, fraction bars, fraction circles, and Cuisenaire rods help students understand the concept of fraction independent of the physical representation. In addition to using manipulatives for new concepts, activities should be oriented to help students connect between concrete, pictorial, and abstract representations of ideas.

However, the use of manipulatives should not become an end in itself. Learning the motions of modeling addition and subtraction with Cuisenaire rods does not guarantee understanding of the mathematical relationship between these inverse operations. It is important not only that the concrete manipulation of materials closely matches the mathematical concept being developed but that the actions are accompanied with appropriate questioning by the teacher and reflection by the student.

In the instructional uses of both technology and manipulatives, the goal is to enhance mathematical thinking. Again, the teacher's role as questioner and decision maker influences the effectiveness of the incorporation of these tools.

Essential Elements of Instruction

Essential Elements of Grade 3 Mathematics with Sample Learning Objectives and Sample Clarifying Activities

The State Board of Education in 1989 revised the essential elements of instruction for mathematics, Grades 1-8. These revised essential elements follow closely the recommendations made by the National Council of Teachers of Mathematics in its nationally recognized *Curriculum and Evaluation Standards for School Mathematics*. According to the Texas Education Agency (1989), "The mathematics curriculum review committee and the Agency [TEA] have tried to be sensitive to a balance between changes expected of teachers and improvements necessary to help students learn mathematics more effectively." Some of these major changes include:

- narrowing the spiral of the curriculum—beginning some topics later and finishing some topics sooner in the curriculum to eliminate some of the redundance
- revising the role of review in the curriculum so that the majority of each grade level is new material and so that review is placed in relevant contexts
- emphasizing the development of problem-solving skills in relevant and interesting situations
- incorporating calculators and computers throughout all grades as problem-solving tools
- adding an essential element on patterns, relations, and functions
- separating the teaching of operations and computation so that all students learn the meaning of the operations
- strengthening the areas of probability, statistics, and geometry
- emphasizing the importance of communication in mathematics

- building on a sound foundation of concepts rather than on rote procedures
- putting mathematics into meaningful contexts

The following essential elements and descriptors for third grade mathematics are annotated with sample learning objectives and sample clarifying activities (except for EE1: Problem-Solving). The learning objectives give a brief look at the developmental components of the concept(s) in the descriptor. The sample clarifying activities are addressed to the teacher and provide a glimpse of what student engagement with this concept might look like in the classroom.

Each set of sample learning objectives and sample clarifying activities is meant to be viewed as an integrated whole (not necessarily matched one-to-one) to clarify the descriptors and to identify connections among them, as well as connections to meaningful problem situations. The Problem-Solving strand therefore is annotated only with sample learning objectives and is connected to the other strands through the language and situations used in their sample clarifying activities.

Many of the activities involve the use of manipulatives and common materials such as hundreds charts or grid paper. A list of these manipulatives can be found in the References and Resources. Also in the References and Resources are lists of the children's trade books, teacher resource books, and software cited in the activities as examples of instructional materials.

The revised essential elements, sample learning objectives, and sample clarifying activities for Grade 3 are:

(1) **Problem Solving.** Experience in solving problems designed to systematically develop students' problem-solving abilities through a variety of strategies and approaches. The student shall be provided opportunities to engage in the following types of activities:

(A) develop an organized approach to solving application and nonroutine problems appropriate for Grade 3;

Sample Learning Objectives

Involving patterns, relations, and functions; number and numeration concepts; operations and computation; measurement; geometry; probability, statistics, and graphing

(B) analyze problems by identifying relationships, discriminating relevant from irrelevant information, sequencing, observing patterns, prioritizing, and questioning;

Sample Learning Objective

Make inferences and predictions

(C) communicate an understanding of a problem by describing and discussing the problem and recording the relevant information;

Sample Learning Objectives

Demonstrate creative thinking through fluency, flexibility, elaboration, creation of new ideas, and spontaneity

Estimate outcomes including appropriate units for outcomes

(D) select appropriate strategies from a variety of approaches;

Sample Learning Objectives

Such as: acting it out; making a model; drawing a picture, guessing and checking; making a diagram, chart, or graph; finding a pattern; using a simpler problem; working backwards; etc.

(E) select appropriate materials and methods for solutions; and

Sample Learning Objectives

Such as: concrete manipulatives, mental computation, paper and pencil (pictorial and/or symbolic), calculator, or computer

Reflect on the problem-solving process and solution of a problem by evaluating outcomes for reasonableness (including appropriateness of units), make revisions as needed, describe and discuss the process and solution, and make a decision based on the solution

(F) generate and extend problems.

(2) **Patterns, Relations, and Functions.** Use of models and patterns to develop the concepts of relations and functions. The student shall be provided opportunities to:

(A) predict additional terms in a given pattern, describe how the pattern is created, and extend the pattern;

Sample Learning Objectives

Predict additional terms in and describe and extend an arithmetic pattern.

Sample Clarifying Activities

Have students work in pairs. One partner enters a constant function on the Math Explorer calculator, then gives the calculator to the other partner who presses the Constant Function Key and records the resulting pattern. Based on the recorded pattern, the student guesses the mystery constant function that has been entered and predicts what would come next.

Predict additional terms in and describe and extend a geometric pattern.

Have students work together as a whole class to construct a square using four toothpicks, then construct four squares using 12 toothpicks. Ask, "What's the greatest number of squares you can make with 20 toothpicks?" Have students make a chart depicting the number of toothpicks squares and a sketch of each figure.

Have students use a hundreds chart and a supply of counters to explore the chart as a whole class and have them describe what patterns they see. For example, a student might say, "I see a pattern in the third row: 31, 32, 33, . . ., 40. The ones digit goes up by 1 and the tens digit stays the same until you get to 40." Record the patterns on the board until no more can be found. Have students place counters on the multiples of 3: 3, 6, 9, ... and describe the pattern. Ask students to close their eyes while you move one or more counters on the display chart, then open their eyes and tell which counters were moved. Have them explain how they know. Have students explore other multiple patterns and consider ideas such as "Describe spatial patterns that result when counters are on multiples of 11. If you place a counter on all the squares where the Digit 5 occurs, how many of the 100 squares will have counters on them? What does the pattern look like? Repeat with other digits. Explore other patterns: 7, 16, 25, 34, 43, 52, 61 . . ., 8, 14, 20, 26, 32, 44, 50 . . ., 9, 13, 17, 21, 25, 29, 33, . . ., etc. Describe the patterns spatially and numerically." See also TEA Module 32, Grades 3-6, Patterns, Relations, and Functions.

(B) record the pairing of members of two sets, determine the relationship between each pair, and use the rule to generate additional ordered pairs;

Sample Learning Objectives

Identify two related sets of information.

Make a chart showing each element of one set paired with a related element from the other set.

Organize the pairs in the chart to illustrate the pattern, or relationship, between the sets of data.

Sample Clarifying Activities

Have students develop a chart relating numbers of insects to the number of legs. Draw a chart similar to a T diagram. On one side, draw insects such as flies from 1 to 10. On the other side, calculate the total number of legs for each number of insects. EX. One fly = 6 legs; two flies = 12 legs; etc.

Make a chart showing each element of one set paired with a related element from the other set.

Organize the pairs in the chart to illustrate the pattern, or relationship, between the sets of data.

Use the identified pattern (relationship, rule) to generate new ordered pairs to extend the chart and make predictions.

Have students make a chart relating batches of cookies to be made to the number of eggs needed, then describe the relationship between the pairs of numbers and extend the chart using the rule.

Present the following situation: "A special machine takes a number, works on it, and sends a new number out the other side. What is the machine doing to the input numbers?" Have students represent input-output pairs by creating a chart, reading them as ordered pairs (input-output), and plotting them as points on a coordinate graph. Students can invent an input-output machine and ask their classmates to discover the pattern for its output and to draw a corresponding graph.

(C) explore patterns of whole numbers, tenths, and hundredths using concrete and pictorial models.

Sample Clarifying Activities

Have students use dimes to represent tenths of dollars and pennies to represent hundredths of dollars. With a calculator, students can add one-tenth repeatedly as dimes are accumulated in a pile and observe the pattern of the display. They can then watch the pattern in the display as pennies are accumulated and the hundredths are added. Have students describe the patterns they notice to their partners.

Have students investigate how many 8s are written down if they write all the whole numbers from 1 to 100 Ask students questions such as, "How could you figure out the answer without writing all the numbers and counting? Explain how you solved the problem."

(3) **Number and Numeration Concepts.** Concepts and skills associated with the understanding of numbers and the place value system. The student shall be provided opportunities to:

(A) read, write, and use large numbers to describe meaningful situations;

Sample Learning Objectives

Use concrete models to demonstrate place value concepts.

Sample Clarifying Activities

Have students estimate the quantity of jelly beans in a jar, then count the jelly beans using place value (groups of 10s, 100s).

Have students work in groups of three to find three different ways to model a number with base ten materials.

Ask students questions such as "Given 124 beans, how many sets of 10 can you make?" Students should predict, then verify with the beans.

Read and write stories involving large numbers and use models to explore their relative magnitudes.

Toss a die 10 times. After each toss, have students place the digit tossed in one of the three possible columns (ones, tens, hundreds) on a place-value chart. After 10 tosses, students can add to find their totals. The winner is the player who is the closest to 1000 without going over.

(B) compare and order whole numbers;

Sample Learning Objectives

Devise strategies to estimate numbers from visual representations.

Use concrete models to compare and order whole numbers from greatest to least and least to greatest, using vocabulary such as "greater than," "less than," "equal to," and "between."

Use patterns to devise strategies to compare and order whole numbers.

Read and write sentences using the symbols <, >, or =.

Sample Clarifying Activities

Show students a transparency for five seconds with 50-60 objects scattered over it and ask them to quickly estimate the number of objects they see. Have students share their strategies for estimating; for example, looking at a small group of five or six and estimating how many of those groups are on the page.

Have students use play money bills to model prices of comparable objects from different catalogs. Students can devise strategies for determining which item costs the most or the least and share their strategies with the class or record them in their mathematics journals using appropriate symbols.

(C) make generalizations about how to determine oddness and evenness of numbers;

Sample Learning Objectives

Record information from concrete experiences with odd and even numbers.

Identify patterns and use them to make generalizations about the properties of odd and even numbers.

Sample Clarifying Activities

Have students investigate which numbers can be distributed evenly between two boxes and write a description of the patterns discovered.

Have students investigate which numbers can be represented by two even rows of dots or cubes and write a description of the patterns discovered. *Madeline* by Ludwig Bemelmans is a good book to enhance the idea.

Have students investigate sums of consecutive odd numbers with symbols, pictures, and objects. Students can record findings in an organized chart and look for patterns.

Have students use calculators to find numbers that multiply to form the magic product 120 and satisfy these conditions: two even numbers, two odd numbers, one even number and one odd number, three even numbers, three odd numbers, two even numbers and one odd number, two odd numbers and one even number. Ask questions such as "Why are some of the problems impossible? What strategies did you use to find the numbers? Find the numbers that multiply to form a new magic product of 315 and satisfy the previous conditions."

(D) use symbols to record fractional names for concrete models of whole objects or sets of objects;

Sample Learning Objectives

Connect fraction symbols to fraction vocabulary developed in previous activities.

Compare the meanings of the numbers in the numerator and the denominator.

Read and write fractions for halves, thirds, fourths, sixths, eighths, and tenths.

Sample Clarifying Activities

Have students use colored tiles or paper to make three candy bars and figure out how to share the candy so everyone in their group of four gets an even share. Introduce appropriate fraction symbols as they are needed by students in their descriptions.

Have students grab a handful of two-color counters, drop them on the table, and identify the fraction of the set that is red and the fraction of the set that is yellow. Students can name the fractions orally and write them in symbol form, then look for patterns in the numerators and denominators of the pairs of fractions.

(E) use concrete models to compare fractional parts;

Sample Learning Objectives

Use concrete models to compare fractional parts, connect the concrete models to the appropriate fraction symbols, and look for a pattern.

Use the patterns to devise a plan to compare fractions.

Write number sentences using the symbols <, >, and = to compare halves, thirds, fourths, sixths, eighths, and tenths.

Sample Clarifying Activities

Have students make a fraction kit using seven different colored strips of paper, all the same size with one color identified as the whole; students can fold and cut the other strips into various fractional parts (halves, thirds, fourths, sixths, eighths, and twelfths) and label them with appropriate symbols. Have students use the kit to compare different fractional parts.

(F) **use concrete models and pictures to represent different names for the same fractional part;**

Sample Learning Objectives

Connect different concrete models of equal amounts to appropriate fraction symbols.

Look for patterns in the different fraction symbols for equal amounts.

Use the patterns to devise a plan for generating different names for the same fractional part.

Identify equivalent fractions for halves, thirds, fourths, sixths, eighths, and tenths.

Sample Clarifying Activities

Have students use their fraction kits or pattern blocks to find fractions that are equal and record their different names and symbols. Students can organize their results to generate conclusions about equivalent fraction symbols: 1/2 = 2/4 = 3/6 . . ..

Given a geometric shape (rectangle, circle, triangle), students will fold it in half and mark the fold with one color. Students can then fold the halved shape some number of times in equal parts, say in thirds, and mark these folds with another color. When the shape is opened, students can observe and record that 1/2 = 3/6. Students can then predict equivalent fractions (e.g., 3/4 = 6/8) and confirm by folding.

(G) **demonstrate various collections of coins and bills that equal a given value;**

Sample Clarifying Activities

Give students a value of dollars and cents and various play coins and bills to use to construct that amount. Have students compare their combinations of coins and bills to those of other students. Who used the most coins? Who used the least coins? Who used the most bills? Who used the least? Students can arrange the different combinations into a chart and look for patterns.

Have students explore the following problem: If a customer pays for a purchase that costs $4.25 with a $5.00 bill, how much change should the clerk give the customer? How do you know how much change should be given? What are all the different combinations of coins that could be used to give this change? What are all the possible combinations of coins you could use to give the customer change if the customer refuses to accept any nickels?

Ask students to explore ways to make $5.00 using exactly 50 coins. Have them verify why they think they have found all possible solutions.

(H) write the value of a given amount of money in words; and

Sample Clarifying Activities

Have students set up a banking situation in the classroom where checks are used instead of cash and money amounts must be written on the checks correctly in words before the checks are valid.

Have students write story problems involving money using words rather than numbers. Students can trade and work each others' problems.

(I) develop place value concepts of tenths and hundredths using physical models.

Sample Learning Objectives

Use money to extend place value concepts to tenths and hundredths.

Use concrete models to develop place value concepts to tenths and hundredths.

Identify, read, and write decimals to tenths and hundredths in meaningful situations.

Sample Clarifying Activities

Have students investigate the characteristics of a decimal square (a 10 x 10 grid) as representing one. Students can divide the grid into 10 equal parts and develop the concept of tenths, then divide the grid into 100 equal parts and develop the concept of hundredths. Have students compare tenths and hundredths.

Have students place a penny on each one of the hundredths spaces of the decimal square. As students trade lines of 10 pennies for dimes, they can mark Xs in the places on the decimal square that were covered. This activity may be extended to exchanging dimes for dollars. Students can record the amounts and connect the decimal places in the written amounts to the number of pennies and dimes and the amounts of area shaded in the decimal square.

Have students measure lengths using a meter stick. Connect decimeters to tenths of the meter stick and centimeters to hundredths of the meter stick. Students can record their measurements in tenths and hundredths of meters; e.g., "The blackboard is 2.58m long means that it is 2 whole meters, 5 decimeters, and 8 centimeters long."

(4) **Operations and Computation.** Use of manipulatives to develop the concepts of basic operations on numbers and to apply these concepts to the computational algorithms. The student shall be provided opportunities to:

(A) demonstrate with concrete models the properties of multiplication (identity, commutative, associative);

Sample Learning Objectives	*Sample Clarifying Activities*
Use manipulatives to demonstrate the commutative property of multiplication.	Have students use assorted counters (buttons, cubes, beans) to explore the concept that 3 sets of 2 equals 2 sets of 3. Students can explain their discoveries to their partners and predict other similar combinations. Have students repeat the activity using array patterns.
Use manipulatives to demonstrate the associative property of multiplication.	Have students use interlocking cubes (or blocks or sugar cubes) to explore the concept that six 3 x 4 layers [(3 x 4) x 6] would be the same as three 4x6 layers [3 x (4 x 6)]. Students can explain their discoveries to their partners and predict other examples.
Use manipulatives to demonstrate the multiplicative identity.	Have students generate stories using the multiplicative identity (1 group, or 1 in each group) and model the story with concrete objects. Students can discuss the results of multiplying with 1.

(B) explore multiplication and division using a variety of different models, and use patterns from explorations and properties of whole numbers to generate multiplication facts;

Sample Learning Objectives	*Sample Clarifying Activities*
Using a variety of concrete models, join sets of equal groups and break sets apart into equal groups to illustrate multiplication and division.	Have students use small paper cups or egg cartons cut into different arrays (6 cups; 10 cups; 2 cups) to explore different arrangements of equal numbers of objects. Example: Put 2 beans in each of the 6 cups. How many beans are there in all? (6 x 2 = 12) Example: Take 21 marbles and separate equally into 5 cups. How many marbles can be put in each cup? 21 ÷ 5 = 4 in each cup with 1 left over. Have students discuss the meaning of the extra marble.
Describe orally and in writing observed patterns in and connections between multiplication and division.	Have students brainstorm all the things they can think of that come in sixes, sevens, eights, and nines and record them in chart format. Students can use the information on the class chart to write story problems, illustrate them and write the answers on the back. Post the problems for others to solve.

Have students read the story *The Doorbell Rang* by Pat Hutchins and act out stories using cookies, solving various multiplication and division problems. Given a number sentence like (4 x 2 = __), students can then write a cookie story problem.

Have students work on the following problem: Four children were walking to school. On the way, they found a five-dollar bill. When they got to school, they told their teacher. She said that the principal would probably be the person who would find out who lost it. The children gave the five-dollar bill to the principal. A week later, the principal called the four children into her office and told them no one had claimed the money. "It's yours, she said, but you'll have to share it equally among the four of you." Give each group of four students a five-dollar bill (play money) to figure out the solution. They may exchange the $5 for other bills and coins, but they must ask for exactly what combination they want and it must equal $5. Students can discuss other questions such as "Suppose the children had found 50 cents instead of a five-dollar bill? How would they share the amount?"

Following the story *Patchwork Quilt* by Valerie Flournoy, have students generate all possible rectangular quilts for 12 squares (color tiles). Ask students questions such as "How many different quilts could you make if you had 13 squares? Which numbers of squares would make better quilts?"

Provide the students with a bucket of cubes; e.g., 20. Ask students, "How many different ways can you divide these into equal groups? Record your findings with division number sentences. How do you know you have found all of the ways?"

Identify patterns on a multiplication facts chart.

Use the patterns in the multiplication facts chart to generate multiplication strategies.

Have students fill in the products in a blank multiplication facts chart. Students can then share any strategies or shortcuts they used in completing the chart.

(C) illustrate the connection between concrete materials and the subtraction algorithm, and use the subtraction algorithm to subtract numbers with and without regrouping;

Sample Clarifying Activities

Have students work in pairs using base ten materials to explore subtraction of two- and three-digit numbers with and without regrouping. Partners explain to each other what they are doing and record their actions. Through discussion, connect the students' actions to written subtraction algorithms.

Working in pairs, each student can start with a base ten hundreds square, subtracting quantities according to the roll of 1 or 2 dice. Partners take turns and must agree on the subtraction procedures. The person who reaches 0 first wins.

(D) solve problems using addition and subtraction facts and algorithms, using a calculator with large numbers and/or with many addends;

Sample Learning Objectives	*Sample Clarifying Activities*
Create word problems to illustrate addition/subtraction with and without regrouping.	Give a headline such as "57-29," and have students work in groups to write original story problems that illustrate the headline. Groups can exchange problems with other groups.
Determine an appropriate algorithm to solve a given word problem.	Have students use a restaurant menu to create and solve problems such as "If you take a family out, how much will it cost? If you order certain items, how much would it cost? If you have $25.00, what could you buy?" Students can choose when a calculator would be helpful.
Select the appropriate operation to use with the calculator to solve addition/subtraction involving over three addends or numbers over three digits.	Give each pair of students a calculator and ask the students to enter 699. Have them take turns drawing cards labeled with two- and three-digit numbers. After each draw, the number on the card is subtracted from the number shown on the calculator. Students keep a record of the numbers subtracted and a tally of the number of subtractions it takes to reach or pass 600. Students can use estimation of their lists to check if the display on the calculator is reasonable. Once the students reach or pass 600, they can repeat the activity beginning at another number; e.g., 899. They then can compare the number of subtractions needed each time.

Assign each student a two-digit number for the title of a mathematics booklet. Have students create as many subtraction questions as they can for which the answer is the number title. Students can read one another's final booklets and check the responses.

(E) determine whether a given problem can best be solved using estimation, pencil and paper calculation, or a calculator;

Sample Learning Objectives

Identify problems that can be best solved by paper and pencil calculations.

Describe and create problems that can best be use of the calculator.

Use estimation skills to confirm the reasonableness of an answer when using a calculator.

Sample Clarifying Activity

For homework, have students interview family members concerning all the ways they used mathematics during the past week. In class, students can role play the different situations where computation was necessary and justify their choices of the most appropriate method of solution (estimation, pencil/paper, calculator).

(F) use properties of operations and problem-solving strategies to do mental calculations with addition and subtraction, extending beyond fact recall; and

Sample Learning Objectives

Experiment with properties of numbers and operations to develop mental strategies for adding and subtracting.

Mentally compute given oral problems and justify the strategies used.

Mentally compute reasonable answers for given written problems and justify the strategies used.

Sample Clarifying Activities

Create a set of game cards in matching pairs (first card, 8 tens and 4 ones; second card, 7 tens and 14 ones). Shuffle the cards and place them face down one at a time. The first player turns over two cards. If the cards match, the player keeps them. If the cards do not match, the cards are turned back over and play moves on to the next player. Play continues until all matches are made.

Give students computational situations like 22 x 5 that are not basic facts and encourage them to devise a strategy to do the computation. For example, "Ten 22s would be 220, so five 22s is half of that or 110." Have students share their various strategies with the class for discussion and agreement or disagreement.

(G) add and subtract money using models;

Sample Clarifying Activity

Have students bring items from home to create a classroom store or use pictures of items cut from catalogs to create pages of items labeled with prices. Have students work in partners using play or real money to purchase items and record the sums for their purchases and the differences for change received.

(5) **Measurement.** Concepts and skills using metric and customary units. The student shall be provided opportunities to:

(A) find the area of a figure by covering it with nonstandard units of area;

Sample Clarifying Activities

Have students estimate and calculate the area of their shoes using beans, centimeter cubes, and inch graph paper. Students can compare their three measures of area and make conclusions about using different area units.

From the tangram set, let the small triangle be equal to 1 unit of area. Have students write their estimates for the areas of each of the other shapes, then use the small triangle to find the actual area of each shape. Have the students construct other polygons, estimate, and find their areas.

Challenge the children to plan a name crossword; e.g., 10 names of classmates, using the smallest area possible. Students can work in pairs and discuss strategies for limiting the area used (e.g., using one letter in two names).

(B) explore the concept of perimeter using standard and nonstandard units;

Sample Clarifying Activities

Have students wrap string around a box to measure its perimeter and compare to other boxes.

Have students use centimeter paper to draw different rectangles that have a perimeter of 24 centimeters. They can use the squares to find and record the area of each rectangle drawn and to look for patterns in the areas versus constant perimeter.

Have students use centimeter paper to draw different geometrical figures with perimeters of 30cm and record the area of each shape. Students can display, categorize, and compare the figures to find patterns in the shapes versus their measurements.

Have students work in small groups to estimate, then measure and calculate the perimeters of the chalkboards and bulletin boards in the room. Students can then design and make borders to trim them.

(C) identify concrete models that approximate weight/mass units for ounce and gram;

Sample Clarifying Activity

Have students make a simple balance scale and locate objects in the classroom, outside, or at home that weigh about one ounce (or one gram). Display the lists of objects on wall charts for students to use as concrete references.

(D) estimate and measure the weight/mass of an object;

Sample Learning Objectives	*Sample Clarifying Activities*
Using knowledge of a concrete model of a standard mass unit, estimate the weight/mass of objects.	Have students fill a plastic grocery bag with objects until you think the bag weighs one pound. Ask questions such as "How will you check your estimate? How would you adjust your choices of objects to get closer to a pound? Suppose that we changed the unit from a pound to a kilogram. Will you need to remove items from your bag or add items to the bag to get as close as possible to one kilogram?"
Weigh objects using a balance or spring scale and standard units.	Have students hold various objects in their hands and estimate how many nonstandard or standard units would be needed to balance them. Students can then use the balance scale to check.
Predict the order of objects from lightest to heaviest, then test the prediction with measurements in standard units.	Have students compare weights/masses of washers, bolts, pieces of chalk, ping pong balls, foam packing peanuts, etc. and order the objects by weight/mass. Students can make observations about size of objects compared to weight/mass (no definite correlation).

(E) estimate answers and solve application and nonroutine problems involving length, weight, and time;

Sample Clarifying Activities

Give students a recording sheet with a list of measurements on it; e.g., 3 ounces, 15 decimeters. Have students look around the room and write down the names of objects estimated as being the measurements on the recording sheet. Students can then measure the objects, record their actual measurements, and record the difference between the actual measurements and the given measurements. For a game activity, have students total their differences to find their scores. The smallest score wins.

Ask students, "How many drinking straws do you think will be needed to equal your height? Write down your guess. Have a partner help you use your straw to measure and record your height. What else would you like to measure? Be sure to estimate first. If the unit changed from a drinking straw to a new crayon, how would your results change?"

Purchase two different packages of raisins—a 15-ounce box and a package of 1/2 ounce mini-snacks. Record the prices you paid on the board. After a whole-class discussion, students can work in small groups to figure out which purchase is a better buy. Have each group present its solution and justify its answer.

Have students practice drawing different stars and choose one star for a mathematics experiment. Time one minute while the students draw as many stars as possible. On sticky notes, have students write their names, draw examples of their stars, record the number they drew in one minute, and post the note on the board. Discuss with the class how they might organize the data. Ask questions such as "If we drew for only half a minute, about how many stars do you think the entire class would draw? Explain how you arrived at your estimate. If we do the experiment again and everyone draws the same five-pointed star, about how many stars do you think the class would draw in all? Explain how you used the evidence on your graph to make your predictions."

(F) tell time on digital and traditional clocks; and

Sample Learning Objectives	*Sample Clarifying Activities*
Identify time as later or earlier than a given time.	Have students relate the time of day to daily activities by making diagrams or charts.
Change the given time on a clock according to intervals of an hour, half hour, five minutes, or one minute.	Have students display the starting and ending times of their favorite television programs or videos.
Name a given time in more than one way. Create and solve story problems involving time, including elapsed time.	Students can make up their own special day of TV programs by charting a time schedule of favorite shows for every hour or half-hour. (Some students may be familiar with networks that begin shows at five minutes after the hour.) These charts may be displayed in the classroom. Some students may also wish to construct a graph of favorite programs.

(G) read a thermometer and describe temperatures related to everyday situations in both Celsius and Fahrenheit.

Sample Clarifying Activities

Have students measure the temperatures of iced tea, hot chocolate, water from a drinking fountain; graph the results; and make summary statements.

Have students measure and record outdoor and indoor temperatures at different times of the day and record the temperatures on a coordinate graph.

(6) **Geometry.** Properties and relationships of geometric shapes and their applications. The student shall be provided opportunities to:

(A) identify characteristics of two-and three-dimensional figures;

Sample Learning Objectives	*Sample Clarifying Activities*
Identify and classify likenesses and differences of two- and three-dimensional figures.	Have each student make a shape on a geoboard. Sort the shapes into two groups according to a secret geometric attribute. (Examples: closed figures vs. not closed figures; figures with square corners vs. figures with no square corners.) Students can analyze the shapes in each category, write a summarizing statement about each category, share their statements, and discuss them.

Identify, count, and record the number of faces, corners, and edges of two- and three-dimensional objects.

Have students classify geometric solids (prisms, cones, cylinders, pyramids, spheres) according to characteristics such as numbers of faces, kinds of faces, numbers of edges, numbers of corners (vertices)

Have students sort two-dimensional figures (tangrams, pattern or attribute blocks, die-cut shapes) according to their attributes. Students can use yarn, string, or other materials to create set boundaries for a Venn Diagram.

Create a Venn Diagram with two interlocking circles. Choose a label for each circle and place the labels upside down so students cannot see them. Place shapes in the appropriate places in the Venn Diagram one at a time. Have students discuss attributes of the shapes in each set and try to guess what the secret labels say.

Have students feel a three-dimensional geometric shape that is in a bag. Other students ask questions about the solid that can be answered with "yes" or "no." Questions continue until the students can identify the correct solid or until 10 questions have been asked without the object being correctly identified.

Have students read the poem "Shapes" found in *A Light in the Attic* by Shel Silverstein. Using attribute shapes, students can arrange the shapes according to what is heard in the poem. Reread the poem, giving students the opportunity to check their placements.

Have students read the poem "Two Boxes" found in *Where the Sidewalks Ends* by Shel Silverstein. Ask questions such as "Choose and describe a box. How are they the same? How are they different? Record your observations." Students can work in groups of four, gather two more boxes and explore ways to sort the six boxes into two groups, and record results. Boxes may include cylindrical containers such as oatmeal or orange juice containers and boxes with objects inside of them. Students in each group show the class their categories and have the rest of the class guess how they labeled their sets.

Have students cut tangram pieces from a square: two pairs of congruent triangles, one middle-sized triangle, one square, and one parallelogram. Ask students to use the three smallest triangles to make a square, a rectangle, a trapezoid, and a parallelogram. Then they can use the five smaller pieces (all but the two large triangles) to make the same shapes. Have students repeat with all seven pieces and record their outcomes on a chart.

Have students explore geoblocks and brainstorm a list of their attributes. Students can classify blocks according to at least three attributes and display their different classifications. Have students make jackets for the blocks by tracing their faces. (If the tracings are done on grid paper, surface area can be discussed.)

(B) investigate congruence and symmetry;

Sample Learning Objectives	*Sample Clarifying Activities*
Using spatial reasoning, justify that two figures are congruent.	Have students make a design with pattern blocks and investigate lines of symmetry by placing mirrors at various positions.
Identify lines of symmetry in various figures.	Have students use folding to determine lines of symmetry of two-dimensional shapes (square, triangle, rectangle, circle, parallelogram). Students can make a chart of the results.
	Have students use a set of die-cut letters to investigate how many of the letters have one or more lines of symmetry.

(C) construct a solid to match a given solid using cubes;

Sample Clarifying Activities

Display an object made with interlocking cubes. Have students examine the object and replicate it.

Have students use cubes as models of rooms to construct models of houses or space stations that fit within given restrictions (certain height, etc.).

Have students construct the outline of a solid using marshmallows and toothpicks.

(D) describe a three-dimensional object from different perspectives;

Sample Clarifying Activities

Make an object from interlocking cubes and cover it. Give students cubes and clues until everyone's construction is congruent to the hidden object. (Examples: I have 9 cubes on one face. At least two of my edges are 3 cubes long.) Students can make up original riddles.

Have students predict what shadows each geometric solid could make. Using a flashlight and the solids, produce shadows. Have students classify the solids according to the shadows they can make. Students can make generalizing statements. *Owl Moon* by Jane Yolen is a good book to enhance this lesson.

Have students describe their rooms as they would look if they were standing on their ceilings.

(E) investigate angles; and

Sample Learning Objectives	*Sample Clarifying Activities*
Identify angles in a given figure.	Have students work in small groups to investigate questions such as "At what times do the hands on a clock form the largest and smallest angles? At what times do the hands on a clock form the same angles?"
Use manipulatives to compare relative sizes of angles.	Have students use a flashlight and two straight objects (straws, rulers) to construct various angles. With partners, students can construct and compare angles to determine which is largest.
	Have students look for examples of angles in the real world and make a collage of examples of angles.
	Have students estimate the number of degrees in each tangram piece by determining its relationship to the right angle (90 degrees). After writing their estimates, students can use a protractor to confirm their estimates.

(F) identify applications of geometry in the real world.

Sample Learning Objectives	*Sample Clarifying Activities*
Identify geometric figures found in actual objects.	Have students list the different shapes found in their favorite toys and make conjectures about why these shapes were used instead of others. Example: Why would a die be the same as a cube? Why does a tricycle have three wheels?
Examine the usefulness of an object related to its geometric characteristics.	Have students go on a geometric scavenger hunt and organize the shapes found in a bar graph. Students can analyze the graph and discuss which shapes were found the most often and why.
	Have students make a geometry scrapbook by cutting out magazine pictures and labeling them according to corresponding geometric figures.
	Have students explore building shapes. Why are most buildings rectangular prisms? Why have pyramids built by ancient peoples lasted so long?

(7) Probability, Statistics, and Graphing. Use of probability and statistics to collect and interpret data. The student shall be provided opportunities to:

(A) collect and record data on the frequency of events;

Sample Learning Objectives	*Sample Clarifying Activities*
Collect and record experimental data.	Have students flip a coin 50 times and record on a table how often heads or tails comes up.
	Have students open bags of M&M's and record the number of each color of candy.
	Have students examine a die. Ask questions such as "If you were to roll the die 25 times, which number do you think would turn up most often? Write down your guess. Roll the die 25 times. Record each roll. Make a graph of the results. Which number turned up most often?" Spinners may be used instead of dice.
Collect and record data by doing a survey of facts or opinions.	Have students predict the five most commonly used letters in the English language and record the predictions. Students can choose a sentence from a book and gather data about how many times each letter appears in this sentence, then compile individual results in a group graph, and compare results to predictions.

(B) make pictographs and bar-type graphs where each cell represents multiple units;

Sample Clarifying Activities

Have students use the M&M data collected to make a pictograph of M&M colors. Students can plan the design of the pictograph, making decisions like "Each M&M picture represents five candies."

Have students collect information about how many pints of milk the class drinks in a week at school. Students can estimate first and graph their estimates, then use actual milk cartons to create an ongoing pictograph. On Friday, discuss and compare their pictograph and their estimates. Have students translate the pictograph into a bar graph having each block represent two or more milk cartons.

(C) formulate questions and make predictions based on organized data;

Sample Learning Objectives	*Sample Clarifying Activities*
Use the data collected from experiments to generate questions and make predictions.	Have students use the data collected from flipping coins to predict how often heads will come up when the coin is flipped 10 times.
Use survey data to generate questions and make predictions.	Have students use the M&M graph to predict how many of each color will be in a randomly chosen bag.

(D) solve application and nonroutine problems for situations involving graphs;

Sample Learning Objectives	*Sample Clarifying Activities*
Generate and answer questions about the information depicted on a graph.	Have students write questions about the ability of different brands of paper towels to absorb liquid, test different brands, organize and graph the data collected, and use the graphed data to answer as many of the questions generated as possible.
Use experimental data to design a game or determine a winning strategy.	Have students use the spinner data to design a game where each player will have about the same chance of winning. Students must justify their designs to the rest of the class.
Use data from a survey graph to make decisions	Have students generate a Favorite TV Night graph to make decisions concerning which products should be advertised with which programs on TV.

(E) locate points on a grid; and

Sample Learning Objectives

Locate points by giving directions (right 4, up 3).

Sample Clarifying Activities

On the classroom floor or on the ground outside, mark giant axis for a coordinate graph. Have a student stand at the origin—where the coordinates are (0,0). Read out directions such as "to the right 6 units, up 5 units" as the student follows the directions. Students then name their location, "(6,5)." Remember to include directions that contain 0 units to the right or up.

Locate points using ordered pairs of coordinates.

Arrange the class in two teams to play coordinate Tic-Tac-Toe. The Xs and Os are put on the line intersections instead of in spaces. The board is larger, usually 10 by 10. The places where the Xs and Os are put must be given according to their ordered pair names. The teams take turns naming the points for the Xs and Os. A leader keeps a record on the grid of the points called by each team. The goal is to get four Xs or four Os in a row.

(F) build patterns displaying pairings of objects from two different sets.

Sample Learning Objectives

Determine and list possible combinations that could be made by pairing objects from two different sets.

Arrange the lists into patterns.

Use the patterns to predict all of the possible combinations.

Sample Clarifying Activities

Have students determine all the possible combinations that could be worn if you had a white shirt and a blue shirt and solid, striped, and plaid pants and organize the combinations in a pattern to determine if all of them have been found. *Corduroy* by Don Freeman is a good book to enhance this lesson.

Have students determine how many different potted plants can be made if you have four kinds of plants (a tulip, a geranium, an ivy, and a cactus) and three different colors of pots (blue, red, and yellow) and display the plants arranged in a pattern. Students can use the pattern to answer questions such as "How many different potted plants could you expect to make if white pots were also available?"

Have students explore various ways to represent the patterns in their combinations. For example, three kinds of ice cream and two kinds of cones could be displayed in a 3 by 2 array. Students can begin to connect these displays to models of multiplication, 3 x 2 = 6 different combinations.

Texas Assessment of Academic Skills

Focus

The Texas Education Agency implemented the Texas Assessment of Academic Skills (TAAS) testing program in 1990. The program is in effect for the 1990-1995 period. The purpose of the assessment program is to provide Texas schools with an accurate measure of student achievement. The scope of content of the TAAS includes more of the instructional targets delineated in the essential elements than previous state assessments. Every section of the TAAS test contains a certain number of broad objectives. These objectives remain constant from grade to grade because they represent the core concepts that form the basis for a sound instructional progression from Grade 1 through Grade 12. What will differ from grade to grade are the instructional targets—or essential elements that comprise each objective. A portion of this extended set of instructional targets is selected for assessment annually, but not every target is tested every year.

The broadened scope of the TAAS assessment program allows for a different focus, one that addresses the academic requirements of the 1990s. Skill areas that demand little more than rote memorization are de-emphasized, while areas that improve a student's abilty to think independently, read critically, write clearly, and solve problems logically receive increased emphasis. This emphasis is in keeping with current national trends in education, which stress the importance and necessity of teaching students higher order thinking skills.

Domains, Objectives, and Targets

The TAAS features three domains—concepts, operations, and problem solving. Each domain contains objectives that are derived from the essential elements. For every objective, there are instructional targets that describe the kinds of mathematical experiences that will reflect that objective. Each instructional target was taken for the most part directly from the essential elements as delineated in the *State Board of Education Rules for Curriculum*. Each target is defined in behavioral terms appropriate for pencil-and-paper testing.

DOMAIN: Concepts

Objective 1: The student will demonstrate an understanding of number concepts.

(a) Compare and order whole numbers
(b) Use whole number place value
(c) Use odds, evens, and skip counting
(d) Recognize and compare fractions using pictorial models
(e) Translate whole numbers (name to numeral/numeral to name)
(f) Recognize decimal place value (tenths and hundredths; using models)

Objective 2: The student will demonstrate an understanding of mathematical relations, functions, and other algebraic concepts.

(a) Use whole number properties and inverse operations
(b) Determine missing elements in patterns
(c) Use number line representations for whole numbers
(d) Classify objects

Objective 3: The student will demonstrate an understanding of geometric properties and relationships.

(a) Recognize two- and three-dimensional figures
(b) Describe and compare two- and three-dimensional shapes
(c) Identify informal representations of congruence and symmetry

Objective 4: The students will demonstrate an understanding of measurement concepts using metric and customary units.

(a) Use measurement units of time, length, temperature, and weight/mass
(b) Find perimeter

Objective 5: The student will demonstrate an understanding of probability and statistics.

(a) Interpret and use charts, tables, bar graphs, and pictographs

DOMAIN: Operations

Objective 6: The student will use the operation of addition to solve problems.

(a) Add whole numbers
(b) Add money using models

Objective 7: The student will use the operation of subtraction to solve problems.

(a) Subtract whole numbers
(b) Subtract money using models

Objective 8: The student will use the operation of multiplication to solve problems.

(a) Use modeling and patterns of whole number multiplication concepts to generate basic facts

Objective 9: The student will use the operation of division to solve problems.

(a) Recognize modeling of division

DOMAIN: Problem Solving

Objective 10: The student will estimate solutions to a problem situation.

(a) Estimate with whole numbers

Objective 11: The student will determine solution strategies and will analyze or solve problems.

(a) Select strategies or solve problems using addition and subtraction with whole numbers

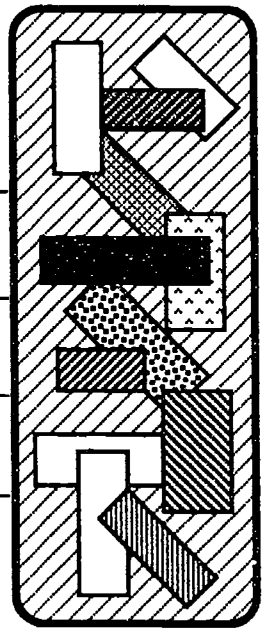
Objective 12: The student will express or solve problems using mathematical representation.

(a) Identify solution sentences
(b) Recognize models for problems
(c) Interpret charts, tables, pictographs, and bar graphs and use the information derived to solve problems
(d) Identify or solve story problems depicting the joining (addition/multiplication) and separating (subtraction/division) processes

Objective 13: The student will evaluate the reasonableness of a solution to a problem situation.

(a) Evaluate reasonableness

Sample Lessons for Teaching Grade 3 Mathematics



The following sample lessons represent the kind of mathematical experiences recommended for students in third grade mathematics classes. These expanded sample activities include ideas for motivational introductions, exploratory questions to ask during activities, summary questions for reflection after exploring the concept, and ideas for extension and assessment. They are included as examples of significant, mathematical tasks that address the state's curriculum requirements in light of the national recommendations. Note that each sample activity involves several essential element descriptors, as well as several objectives from the Texas Assessment of Academic Skills (TAAS). Several different manipulatives are included in these activities. It is important that students use these manipulatives as they work through the activities. Manipulatives and concrete objects enable elementary school students to better understand the mathematical problems and concepts they so often struggle to learn. Students' experiences with manipulatives are recommended in the essential elements, TAAS's instructional targets, and NCTM's *Curriculum and Evaluation Standards for School Mathematics*.

Many activities in this section also recommend that students work together in pairs or small groups. Working together in cooperative groups promotes communication and mathematical confidence and enhances students' problem-solving abilities.

EEs: 2A, 2B *Related EEs: 1A, 1B, 1C, 1D, 4D* *TAAS Objectives: 2, 10, 11, 12, 13*

Objective The student will estimate the number of lines needed to connect groups of dots. The student will record on a chart the actual results of the activity and use the chart to predict future outcomes.

Activity Fancy Flights of Four

Materials Large paper (one sheet per group), paper plates (four per group), tape (one roll per group), thin yarn, hole punch, scissors

Procedure

Introduction:

1. Announce: "Welcome aboard the (teacher's name) Airline. I will be your pilot today. Thank you for flying with us."
2. Pose the problem: If there is one city, how many flight paths would be needed from this city to this same city. (On the board, draw one dot to represent the city.) (0 flight paths)
3. If you have two cities, how many flight paths would be needed to connect the cities. (1 flight path = 1 line segment)
4. Draw two dots on the board. Ask, "How might airlines show flight paths between cities?" Discuss that a straight line is the shortest distance between two dots and the reason the airlines might use this to show their flight paths.
5. Pose the same problem with three cities. Ask, "How many lines would you need to connect each city with each of the other cities?" Lead the students to see that a *T* chart can be used to record the data that have been generated. Discuss how to read and use charts.

no. of cities	no. of paths
1	0
2	1
.	.
.	.
.	.

6. Show students how the data on the chart can be transposed into ordered pairs where the first element of the pair designates the number of cities and the second number designates the number of flight paths: (1,0), (2,1), (3,3). Have students discuss the advantages and disadvantages of each of the two forms of recording the data.
7. Announce: "Our airline is expanding. We would like to have flights to more than 3 cities. Your job is to find out how many flight paths we will need to connect different numbers of cities, starting with 4 cities and going to 10 cities. We will need to give this information to the air traffic controllers so they can regulate the flights. Our goal is to find out how many flight paths (lines) will be needed to connect 10 cities. This will be done by using yarn to connect holes made on our paper plate maps. Each hole made on our paper plate represents a city. One plate should have 4 cities; one, 5 cities; one, 6 cities; and one, 7 cities. Record the number of flight paths on your group data chart, starting with 1 city and working up through 7 cities. Then use this information to predict the number of flight paths needed for 8, 9, and 10 cities."
8. Discuss student questions to determine comprehension of directions.

Exploration:
Allow students time to work within groups. Monitor group progress and discussions by asking the following questions:

- How did you decide where to put the cities (holes)?
- How did you decide if you had connected all of your cities?
- What patterns have you noticed on your data chart or in the set of ordered pairs?
- What predictions can you make about future outcomes?

Extension:

- Draw the dots and lines on paper for 8, 9, and 10 cities to check predictions.
- Using the chart or an organized list of ordered pairs, predict the number of flight paths for 11, 12, 13, and 14 cities. Add this information to the chart and the set of ordered pairs.
- Label the holes as real cities.
- Using a map of the United States, choose various cities. Determine the number of flight paths between cities and draw the paths.
- Use calculators to predict larger numbers of cities/flight paths or to determine mileage between existing cities.

Summary:
Have each group share their charts, lists, and plates and explain procedures and strategies used. Groups can then share their predictions for 8, 9, and 10 cities. Discuss the predictions and patterns that were noticed within the data. As a class, have students generalize rules and patterns using these data. (Extension activities can be shared at this time.) Have the students answer the following questions:

- How were responsibilities within the group divided?
- What relationships did you notice between numbers of cities and flight paths?
- Was there a specific rule that was followed in your predictions?
- Why is it important to be able to predict other elements of a pattern from using a small piece of the pattern?
- What is the importance of making a chart?
- What are the benefits of using the ordered pairs to represent the relationship between number of cities and number of flight paths?
- What are some other real-life examples of charts we use and what information is given by them?
- What are some other examples of sets of ordered pairs that give us important information?
- Would an airline have to have a flight from each city to every other city?
- How else could an airline plan its flights so the airline would not have so many to schedule?

Assessment

Observe student participation and cooperation. Look for students' use of a rationale for generating the chart information, recording the information in ordered pairs, making predictions, and justifying their answers. Check that each group identified a pattern/rule and then used this pattern/rule in formulating predictions.

Objective The student will use patterns to discover rules used to sort a set of numbers into two sets.

Activity Belongs, Doesn't Belong

Materials chalkboard, chalk

Resources *Maths in the Mind: A Process Approach to Mental Strategies* by Ann and Johnny Baker, Heinemann Publishers

Procedure

Introduction:

1. Draw a line across the chalkboard and label the areas above and below the line:

   Belongs:

   Doesn't Belong:

2. Select a rule to separate numbers into two sets (or have a student select a rule). Rules could include: odd numbers, even numbers, factors of a number, prime numbers, composite numbers, multiples of 5s, etc. The aim of the game is for the rest of the players to determine what the rule is by calling out numbers. If the number belongs to the rule, it is written above the line. If not, it is written below the line beneath the last correct number. It is important to keep "doesn't belong" calls in line under the previous "belong" call, as the comparison between the two is often useful.
3. Some players will identify the rule after a few plays. Instead of calling out their guess at the rule , they call "Eureka!" They can then do one of two things: either keep quiet until the rest of the players have identified the rule or, better still, test their rule by suggesting numbers that they feel sure do or do not belong. Example of game with the rule "is an even number":

Belongs:	2	4	10	. . .
Doesn't Belong:	3	7	21	

4. Beware of difficult, nonmathematical rules like, "The numbers in my phone number."

Exploration:

- Do we have any useful information?
- What patterns have you found? How did you find a pattern?
- Is there more than one pattern? If so, can you find a relationship between the two patterns?
- What strategies can you use to discover patterns?
- What kind of guesses do you think give the most information?
- What happens to your strategies as more guesses are made?
- What guesses can you make to test your rule?
- Did finding patterns make finding a rule easier? Why?

Extensions:

- Create a new pattern and have a partner discover the rule.
- Introduce a special sequence of numbers such as the Fibonacci sequence.

Summary:

- Were all the strategies in the class the same? Did any strategies have specific advantages?
- How do you know when you have found a pattern?
- Do you see any relationships between the patterns?
- How did you organize your guesses?
- What did you decide was a good sequence of guesses to make?
- What kind of shortcuts did you use to find the rule?
- How did you test your conjectures?

Assessment

Questions:

- How would you change your strategy for the next time? Why?
- (See also summary questions.)

Observations:

- Were the students thinking about guesses and developing strategies or guessing randomly?
- Were the students organizing their data?

Tasks:

- Write a summary statement of what you learned about making guesses to find the rule.
- Develop another rule for a new game. Share with the class.
- Make a list of all the number properties you learned and what other properties you know about.

EE: *3B* | *Related EEs: 1A, 1B, 1C, 1D, 1E, 3A, 4D, 4F, 7B* | *TAAS Objectives: 1, 2, 5, 6, 10, 11*

Objective The students will use the concept of place value to make decisions in a game.

Activity Make a Hundred

Materials Calculators, base ten materials, one die, one gameboard, one recording sheet per two or three students, overhead base ten materials, transparency of gameboard, and recording sheet, overhead calculator for teacher if available

Resources *Family Math* by Jean Kerr Stenmark.

Procedure

Introduction:

1. If you rolled a die exactly 7 times, what is the largest sum you could make? (Could you make 100?) What if each number represented that many 10s?
2. Explain the game: Players must roll exactly 7 times and place each digit rolled in either the ones or tens place to make a sum < 100, trying to get as close to 100 as possible. (If you had $100 and could buy exactly 7 items, what would you buy?)
3. Model the game on the overhead as a student rolls two or three times:
   - Write the number in the ones or tens on the record sheet (wherever the class decides).
   - Show the amount by laying the base-ten blocks on the gameboard flat.
   - Record the addend on the calculator. (After the first roll, add the addend and show the sum on the calculator.)
4. Have students play the game at least three times, looking for strategies to play better each time.

Exploration:

- How did you decide to place this digit in the ones place? The tens place?
- What does the record sheet keep track of for you that the calculator doesn't?
- What do the base ten blocks show that the calculator and record sheet don't?
- What does the calculator help you do?
- How does your sum affect your strategy as you play?

Extension:

- Play the game with polyhedral dice other than cubes and see if strategy needs changing.
- Make up a set of 7 rolls that would equal exactly 100. How many can you find?
- Revise the game to include the hundreds place and try to make a sum of 1000.

Summary:

- What strategies did you use?
- How does luck affect the game? What can we use to help us deal with luck in a game?
- How did your strategies change as you played more games? How did your strategies change within a game?
- What if you did not have to roll exactly 7 times? What if it could be fewer? What if you could roll more than 7 times?
- How did you use the (calculator, record sheet, base ten model) to help you decide what to do next in the game?
- Is there any game that you played that could have made a sum of 100 if you rearranged the digits? Use your record sheet and calculator to find out.

Assessment

Questions:

- How would you change your strategy for the next game you played? Why? Play two more games to test your strategy. What refinements would you make based on this new information?
- Present a hypothetical game—show sum on calculator, record sheet, and base ten model. Ask students to analyze the game. Do they agree or disagree with the way it was played, what to do next, etc.?

(See also summary questions.)

Observations:

- Were students using base ten materials appropriately?
- Were students recognizing the connections between the three methods of recording?
- Were they using mathematical concepts to design strategies (e.g., probabilities of getting certain outcomes when rolling dice, using subtraction to determine how much is left)?

Tasks:

- Find the game you played that would have gotten you closest to 100 if you had placed the numbers differently.
- Write a summary statement of what you learned from the game.
- Graph scores from first game, second game, and third game to see if strategies improved the range of scores. (Were scores on third game closer to 100 than those on first game?)

RECORD SHEET
for
ADD ON

Game 1

Tens	Ones

Game 2

Tens	Ones

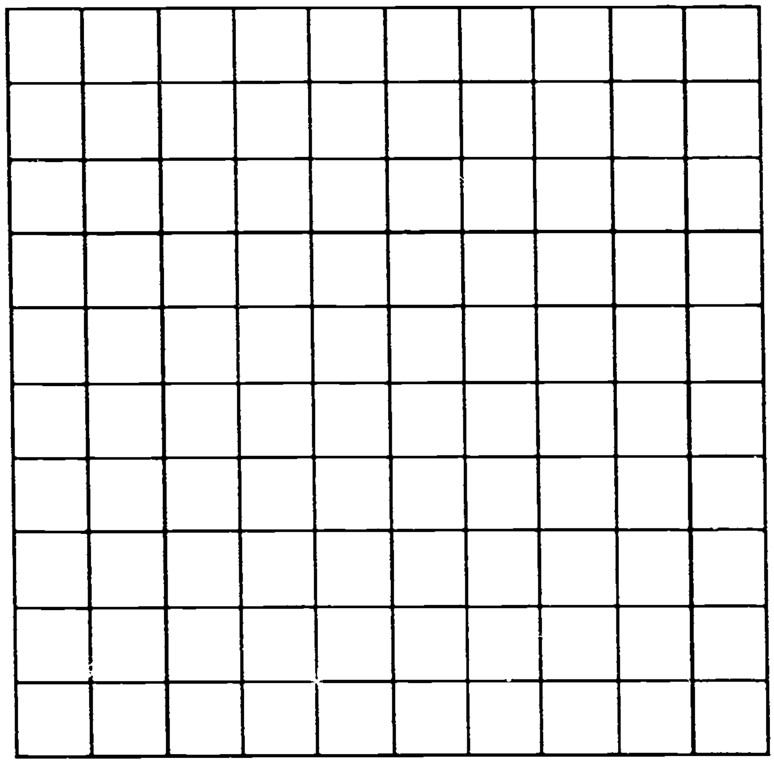
Game 3

Tens	Ones

Game 4

Tens	Ones

HUNDRED GRID



EE: 4B | *Related EEs: 1A, 1B, 1C, 1D, 1E, 2A, 4A* | *TAAS Objectives: 2, 8, 11, 12*

Objective The student will explore multiplication using concrete materials and discover patterns from exploration and properties of whole numbers to generate multiplication facts.

Activity Multiplication Ark

Materials Large sheet of butcher paper, one piece manilla paper per group, glue

Resources *Noah's Ark* by G. E. Haley; *The Shoemaker and His Elves* by C. Brier; *Anno's Mysterious Multiplying Jar* by Mitsumasa Anno

Procedure

Management Suggestions:

- Allow approximately three class periods to complete all activities.
- Set up class in groups of four that can be later put into pairs (Day 3).

Introduction (Day 1):

1. Choose a book that demonstrates the concept of pairs. Read it to the class.
2. Have the students brainstorm all other things that can be grouped into pairs. On the class chart, begin to list students' ideas. For example: eyes, hands, ears, shoes, arms, dance partners, etc.

2s	3s	4s	5s	6s	7s	8s	9s	10s
eyes hands								

Chart can be extended to 12, depending on the students' ability.

Exploration:

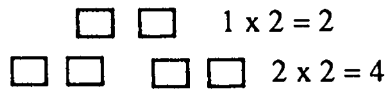
- Group things that can be grouped into 3s. Record two or three responses. Direct students to continue their responses by using the manilla paper to copy the butcher paper chart. They are to start with the 3s and continue through groups of 10.
- Combine ideas onto class chart. Record ideas from each group of students, eliminating any duplicates. Discuss answers, clarify answers. Look at categories that were difficult to fill.
- Have groups meet again and write summary statements about what they did. Have them share the statements with the class.

Extension:

- Using the book *Noah's Ark* as an example, pose the question, "If Noah had loaded the ark with groups that come in ___________" [choose 3s, 4s, etc.], what might have entered the Ark? Draw a picture."

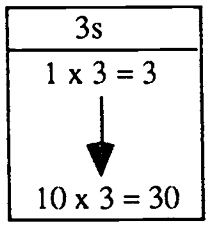
Introduction (Day 2):

1. Review the chart and add any new ideas. Pose a problem taking an example from the chart using the 2s. "If I had six children, how many eyes would there be?" Draw a picture to show the answer. Write the appropriate number sentence: 6 x 2 = 12. Do a few more examples. Demonstrate using the interlocking cubes on the overhead showing the way to discover the 2s facts. Examples:



2. Continue until 10 x 2 = 20. Discuss and clarify.

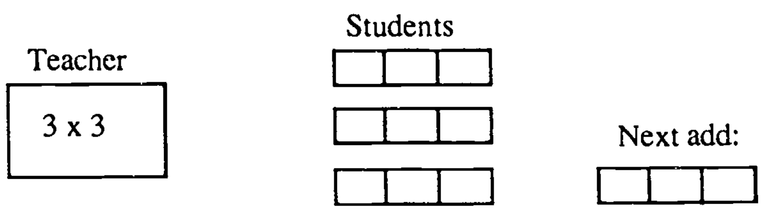
Exploration:
In pairs, students can use the cubes and record their multiplication sentences on paper. Example:



Monitor and assist as needed.

Summary:

- Gather pairs together with their charts and discuss the findings. Put pieces of paper together to show all the facts. Discuss the patterns demonstrated in each number fact. Review the meaning of multiplication as sets of groups (4 groups of 2 is the same as 4x2=8). Send students back in pairs showing them a multiplication sentence. They will create it with cubes. Then have them show what multiplication fact will come next. Discuss the pattern. Example:



- Hold up what they would take away to show what comes before. Discuss the patterns again.

Extension:
Give a list of multiplication sentences (with no answers) out of order and have students arrange them. Put in the answers together. Write a sentence about the pattern.

Introduction (Day 3):

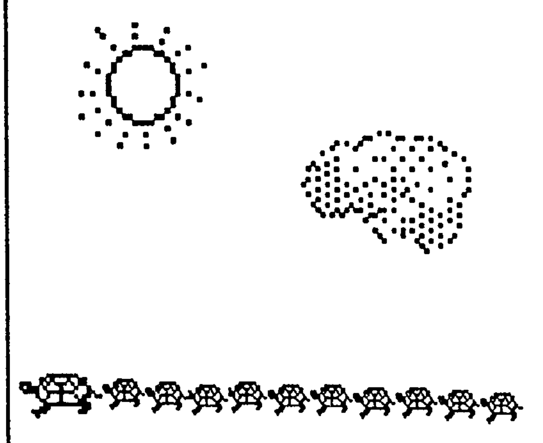
1. Review the multiplication sentences and the patterns. Tell the students that they will be writing stories using ideas from their list of multiplication sentences. Review the charts.
2. Put up a story on the board that ends with a question that generates a multiplication sentence. (Example: Mrs. Jones's third grade class has four sets of twins. The school

wanted to honor all the twins with ribbons. Mrs. Jones was responsible for making the ribbons for her twins. How many ribbons does she need to make?) Write the number sentence 4 x 2 = 8. Have students make up another story that would fit the multiplication sentence.

Exploration:

1. Tell the students that in pairs they will be making pages for a book entitled "Multiplication." Each pair will make one or more pages for the book. The front of the page will be a story problem and an illustration. The back of the page will have the multiplication fact. Example:

FRONT



Once there was a turtle with 10 children.
She wanted to buy them some shoes, but she
didn't know how many shoes to buy.
Help Mrs. Turtle.

BACK

10 x 4 = 40

2. Assign a multiplication fact or facts to each pair of students and hand out paper. Allow time for students to complete their pages. Teacher monitoring and editing will be necessary. Have a pair that finishes first decorate a cover for the book. Put the book together.

Extension:

- Write a story that contains several multiplication situations and have a friend pick them out of the story.
- Put the book in a center and allow students to work the problems in the book and check themselves.

Summary:
Gather together and share the book. Discuss how the stories relate to multiplication. Discuss strategies for discovering appropriate multiplication sentences. Edit if necessary and discuss any changes made if needed.

Assessment

- Teacher observation of student participation during activities and discussion.
- Student-made summaries from Day 1.
- Student-made multiplication sentences from Day 2.
- Student-made pages from the book made on Day 3.

EE: 4D | *Related EEs: 1A, 1B, 1C, 1D, 1E, 2A, 2B, 2C, 3A* | *TAAS Objectives: 1, 2, 6, 10, 11, 13*

Objective The student will identify number patterns, both visually and numerically. The student will also practice addition with regrouping.

Activity Palindrome Patterns

Materials Crayons or markers; hundreds charts

Resources *A Collection of Math Lessons from Grades 3-6: Operations and Computation* by Marilyn Burns

Procedure

Introduction:

1. Review the concept of a palindrome. A palindrome is a number that reads the same frontward and backward such as 44, 252, or 8008. A number that is not a palindrome such as 13 can be changed into a palindrome by adding in a certain way: You reverse the digits and add. Record how to do this on the board. Since it took one addition, 13 is a one-step palindrome. Some numbers take more than one addition. Demonstrate with 68 (a three-step palindrome).

$$\begin{array}{r} 13 \\ +\underline{31} \\ 44 \end{array} \qquad \begin{array}{r} 68 \\ +\underline{86} \\ 154 \end{array} \quad \text{and} \quad \begin{array}{r} 154 \\ +\underline{451} \\ 605 \end{array} \quad \text{and} \quad \begin{array}{r} 605 \\ +\underline{506} \\ 1111 \end{array}$$

2. Present the problem to be solved. Each group is to investigate in this way all the numbers from 0 to 99.
3. Students should decide how to divide up the work within their group and how to keep records of their information. Examples of record-keeping devices include:
   - listing the numbers 0-99 in a chart showing the number of steps and the palindrome reached
   - grouping the numbers 0-99 according to the palindrome they generate
   - grouping the numbers 0-99 according to how many steps it takes to make a palindrome. (Tell students to beware of 98 and 89; they take 24 steps. The palindrome is 8,813,200,023,188.)
   - recording the groups of numbers on a hundreds chart by coloring one-step palindromes red, two-step palindromes green, etc.

Exploration:

- How did your group divide up the work?
- How did your group decide to record your information?
- What shortcuts, if any, have you developed in finding the information?
- Are you beginning to notice any patterns?
- Are 1-digit numbers palindromes? Why or why not?
- What difficulties are you having in finding the palindromes?
- What things have you found to be important in finding the palindromes?
- How might a calculator help you in finding patterns?

Summary:

- Discuss the group processes. Ask groups: "How did you divide up the work? Was your system a good one? Did you change your system during the activity? What do you think would be the best way for a group to do this activity?"

- Ask, "What are some patterns that emerged on the charts?" Have students record summary statements. Some groups notice, for example, that when the sum of the digits of a number is less than 10, the number is always a one-step palindrome. Others notice that all resulting palindromes are multiples of 11. Have them look for other patterns such as these.
- How did you categorize the one-digit numbers—as one-step, already palindromes, or not at all? There is no right answer for this, but groups should be able to explain their decisions.

Extension:
Discover words that would be considered a palindrome such as mom, dad, Hannah, etc. Can students write a palindrome sentence?

Assessment

Questions:
(See summary questions.)

Observations:
- Were there patterns that students assumed were there but turned out not to be a pattern once the addition was completed?
- Did students discover that once you know that 13 is a one-step palindrome, that 31 will also be a one-step palindrome?
- Did students use appropriate strategies for organizing their information?
- Did students in the group work together to plan their organization?
- Did students apply their knowledge of addition to solve the problem?

Tasks:
- Write in your mathematics journal a summary paragraph of what you learned about palindromes.
- Given a number from 0-99, use the chart or table that your group generated to describe its behavior when making a palindrome.

EEs: 5A, 5B *Related EEs: 1A, 1B, 1D, 1E, 1F, 5E, 6A, 6F* *TAAS Objectives: 3, 4, 6, 8, 10, 11, 12, 13*

Objective The student will compare shapes with perimeters of 30cm by categorizing them and looking for patterns.

Activity What Can I Make With 30 Centimeters?

Materials Centimeter grid paper, scissors, tiles, markers

Procedure

Introduction:

1. Give students tiles and have them build a flat shape.
2. Discuss the perimeter and area of the shape.
3. Challenge students to build a flat shape with a perimeter of 16 units. Discuss the areas of the different shapes made. Bring students to the conclusions that different shapes can have the same perimeter and shapes with the same perimeters can have different areas.
4. Pass out the grid paper and ask students to find as many flat shapes as possible that have perimeters of 30 units (cm, if cm grid paper). Discuss the need to follow the lines on the grid paper in making the figures to be able to measure the perimeter easily. Also, students must be able to cut out shapes and have each shape hang together. (This avoids shapes in which squares touch only at their corners.)
5. Have students post their figures in a display area. Discuss the desirability of posting only the ones found that are different from the ones already in the display.

Exploration:

- Did you use the same number of squares each time to make a shape with a perimeter of 30 units?
- How do you know the perimeter of this shape is 30 units?
- How do you know your shape is the same as (congruent to) another student's?
- What strategies are you using to find your shapes?
- What patterns have you found in the shapes?

Extension:

- Can you make your initial with a 30cm perimeter? On the grid paper? What if you had a piece of string 30cm long?
- If you had a piece of string 30cm long, how could you use it to find shapes with 30cm perimeters? Would you still have to follow the lines on the grid paper? Find some other shapes using the string.

Summary:

- Select some of the categories and organize the shapes to look for patterns.
- Lead students to classify the shapes according to area and look for patterns.
- Do all the shapes with 30cm perimeters have the same area?
- Do some of them have the same area?
- What is similar about the shapes with the smallest area compared to the shapes with the largest area?
- What statements could you make about using a certain perimeter to enclose the smallest area? The largest area?
- What strategies did you use to find shapes with 30cm perimeters?
- Did looking for patterns make solving the problem easier?

Assessment

Questions:

- How is perimeter measured?
- How is area measured?
- What techniques could you use to change a shape to increase its area while keeping the perimeter the same?
- What techniques could you use to decrease a shape's area while keeping the perimeter the same?

Observations:

- Were students generating shapes with 30cm perimeters?
- Could students justify that their shapes had 30cm perimeters?
- Could students justify that their shapes were the same or different from other ones?
- Could students determine the areas of the shapes using the grid paper?
- Were students discussing techniques for generating the shapes?

Tasks:

- Record summary statements in a journal.
- Write a description of the activity and what you learned from it.
- Make a shape on the grid paper that has a 42cm perimeter and encloses the most (least) area; justify your choice of shape.

EE: 5E *Related EEs: 1A, 1B, 1C, 1D, 1E, 1F, 2A, 2B, 5D, 5F, 7E* *TAAS Objectives: 1, 2, 4, 5, 10, 11, 12, 13*

Objective The student will investigate the relationship between the length of the string of a pendulum and the number of swings in a given time by generating data, recording the outcomes, and looking for a pattern.

Activity Swing in Time

Materials String, cut in lengths of 110 cm; 1 oz. and 2 oz. fishing weights, one of each for each group, labelled 1 and 2, respectively; metric ruler or tape measure; markers; overhead projector; chart paper

Resources *Popping with Power* from AIMS Education, 1987

Procedure

Introduction (Day 1):

1. Bring in objects that have pendulums; e.g., clock, metronome; discuss.
2. Show a teacher-constructed pendulum.
3. Model holding the pendulum, pulling the weight back. Allow the pendulum to swing for 30 seconds while the class counts the swings. Repeat releasing the pendulum at several different angles and discuss the results. (The counts will be approximately the same as long as the length is not changed.)
4. Have students tie a 110cm piece of string to the 1 oz. weight, then measure and mark the piece of string at 10cm intervals beginning at the weight up to 100cm.
5. Instruct students to begin by holding the pendulum at the 10 cm space, count for 30 seconds and record the results; then repeat holding the string at the 20, 30 . . . 50cm marks.
6. Allow students time to observe patterns; chart results on a graph.
7. Have students make a prediction for results occurring at 60cm, 70cm . . . 100cm. (Results should follow the pattern.)
8. Have students record predictions and test.

(Day 2): Repeat the same procedure using weight No. 2.

(Day 3): Review the previous days' concepts, compile graphs into a class chart, and compare.

Exploration:

- How did the different lengths of string affect the number of swings? Did a pattern form?
- Did the weights affect the number of swings?
- How are you recording your data? Why?
- Did you notice a pattern as the results were recorded?
- Did you try different methods of recording?
- Did you develop any shortcuts in your recording? Explain.
- What patterns do you see in the class chart? Is there more than one pattern?
- How can you use a pattern to predict what will happen? How will you test your prediction?

Extension:

- Suppose you want to find a pendulum that will swing 60 times per minute, what length of string will you need to cut?
- Suppose you swing more than one pendulum at one time, in opposite directions, 2 one way and 1 another, criss-cross, etc.
- Predict the amount of time it would take a pendulum to stop completely.
- Construct a different shaped pendulum; for example, a paper cone filled with sand or salt. Allow the cone to swing holding it over a piece of paper. Observe the pattern it makes; record.

Summary:

- What happened when we changed the length of string?
- What patterns did you find?
- What predictions can you make using the patterns you have observed?
- Why is it important to record this information in a clear and meaningful way?
- Why might one way be clearer than another?

Assessment

Questions:

Given a certain situation (e.g., using a No. 3 fishing weight and a 25cm length of string), allow student exploration; have students predict possible outcomes and justify their answers.

Observations:

- Were students using appropriate and clear methods to record their results?
- Did student discussion indicate an understanding of the activity?
- Were students arranging data to help them find patterns?

Tasks:

- Describe the pattern you saw in the graph.
- Record summary statements in a journal.
- Write a description of the activity and what you learned from it.

Sample Graphs

The teacher needs to lead students to develop charts similar to these:

length of string	10	20	30	40	50	60	70	80	90	100
your estimate of swings										
actual no. of swings										

For weight #1

	10	20	30	40	50	60	70	80	90	100
actual no. of swings										
your estimate										

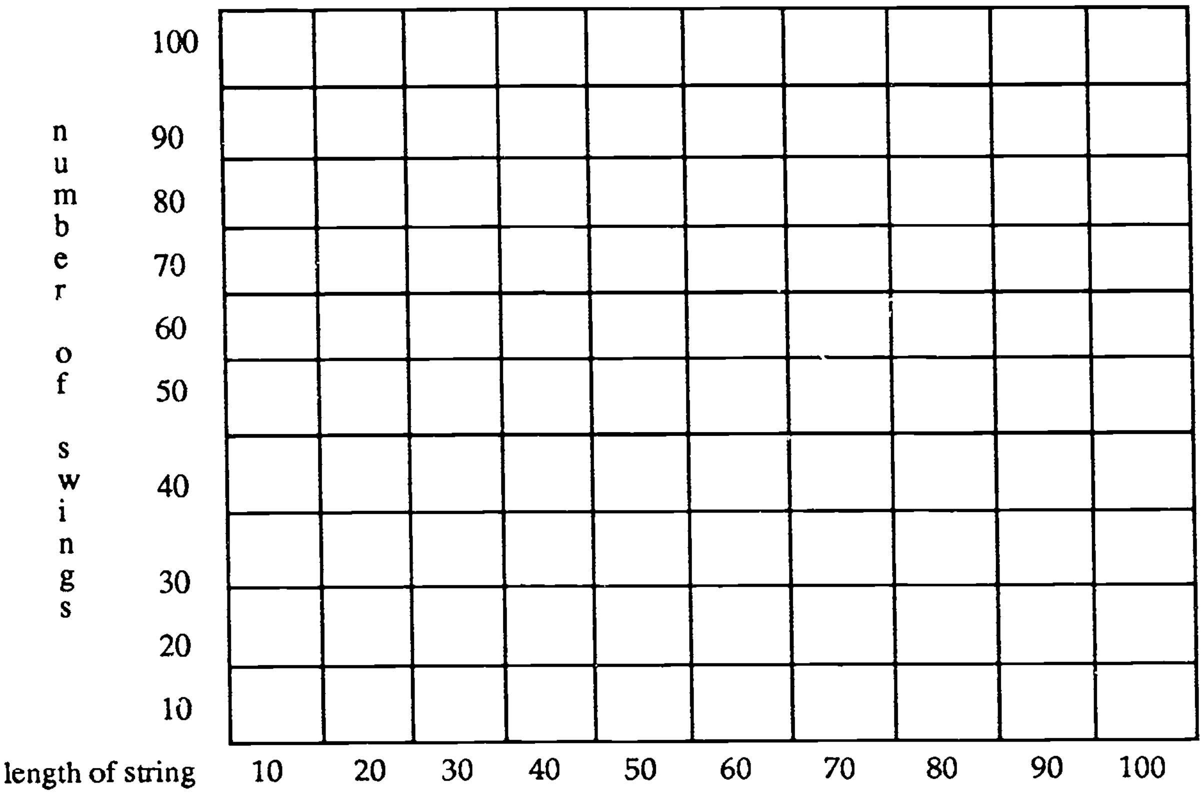
For weight #2

	10	20	30	40	50	60	70	80	90	100
actual no. of swings										
your estimate										

Data in the charts can also be recorded as ordered pairs before making the graph: (length of string, number of swings)

(10,) (20,) (30,) (40,) etc.

Possible class graph/chart:



	Group 1	Group 2
No. 1 weight—	green	yellow
No. 2 weight—	blue	red

EE: 6B | *Related EEs: 1A, 1B, 1C, 1D, 2A, 5A, 5B, 6A, 6D* | *TAAS Objectives: 2, 3, 10, 11, 12*

Objective The student will use one-inch square tiles and paper ruled into one-inch squares to solve the problem of finding all the possible ways to arrange five squares into shapes called pentominoes.

Activity Oh No! My Ominoes!

Materials Overhead color tiles, color tiles, and squared paper

Resources *A Collection of Math Lessons from Grades 3-6* by Marilyn Burns.

Procedure

Introduction:

1. Begin the lesson by telling the students that they will be searching for different shapes that can be made from five squares, using the tiles that have been distributed. Explain that these shapes are called pentominoes.
2. Discuss rules to the activity. (a) When you arrange the squares into shapes, the requirement is that at least one whole side of each square touches a whole side of another. (b) You will have to decide if the shapes you create are the same or different. If two cut-out shapes fit exactly, they are called congruent and count as only one shape.
3. Discuss the derivation of the word *pentomino*. A domino is made from two squares. Discuss trominoes (three-square versions) and tetromino which is a four-square shape. Pentominoes are five-square versions.
4. With the entire class, find all the different arrangements of three squares. What shapes could you make? Allow students to discuss and make shapes. Stress that cutting out shapes will help determine if shapes are congruent because you can flip and turn the cut-out shapes.
5. Once the class understands the procedure, present the problem to be solved in small groups. Each group is to find all the possible ways to arrange five squares into pentominoes. Each group must cut each of them out of the squared paper provided. Each group should make one set of all the different pentominoes, accounting for all possible arrangements.

Exploration:

Groups need to decide when they think they have found all possible solutions. Find out what strategy they used to decide when they think they have found them all. Other groups usually overhear the discussion and offer the total number that they have found. When one group has more, it encourages the other groups to find them.

Extension:

- Start with the shape that looks like the Red Cross symbol. Ask the students to visualize how they could fold up the sides of this shape so that it would be a box without a lid. Predict which side would be the bottom of the box, opposite the open side, by putting an X on the appropriate squares on their group's pentomino. For each of the other pentominoes, have students inspect the pentominoes and predict whether or not each would fold into a box. They can mark an X to indicate the bottom of the box and test their predictions.
- Sort the shapes in several ways. Categorize which shapes have rotational symmetry, line symmetry, and no symmetry.

- Using a 5 x 12 squared sheet as a gameboard, with squares matching the size of the squares used for pentominoes, use the board and try to fit all 12 pieces onto the board. This could be a two-person game, with players taking turns placing pieces on the board. The object is to play the last possible piece so that it is impossible for the opponent to fit in another.

Summary:
- Discuss any strategies that groups used to discover all shapes.
- Allow one group to post its findings so that other groups can compare. For future use, students need to sketch all 12 shapes on centimeter-squared paper.

Assessment

Questions:
- What strategies did you use to find the shapes?
- How did you convince yourself and your group that all the shapes had been found?
- How did you judge whether a shape was new or the same as (congruent to) some other?
- What is alike about all the shapes?
- In what ways are the shapes different?

Observations:
- Did students work together to develop a strategy for finding all the shapes?
- Did students use appropriate geometric vocabulary when discussing the shapes?
- Did students use appropriate strategies for determining whether shapes were different?

Tasks:
- Separate the shapes into two or three groups and explain your sorting rule.
- In your mathematics journal, discuss how your group decided all the shapes had been found.

Objective The student will make predictions and draw conclusions based on sampling and probability concepts.

Activity Pretzel Probability

Materials Paper sacks; markers; cubes (10 each of six colors); stickers

Resources *A Collection of Math Lessons from Grades 3 through 6* by Marilyn Burns

Procedure

Introduction:

1. Inform students that companies frequently include prizes in their products to stimulate sales. (Show Happy Meal or Cracker Jacks.) A pretzel company recently put a sticker in each box of pretzels. It takes six different stickers to make a complete set. The company is striving to be fair, so they are packaging and shipping an equal number of each sticker to stores. When you buy a box, you have an equal chance of getting any one of the six stickers.
2. Ask students to predict how many boxes of pretzels you would have to buy to get a complete set of stickers.
3. Let students discuss predictions in small groups.
4. Afterwards, have them share predictions with the whole class.
5. Ask students to write predictions in their journals.
6. Inform students that because we do not actually have the boxes of pretzels, we will simulate the pretzel purchases. (Mention flight simulators.)
7. Show students a paper sack containing six cubes (one each of six colors). Ask students what is the chance of getting a red cube. Lead into a discussion of probability/chance and fractions.
8. Draw from the sack several times noting the color of the cube each time. (Replace the cube and shake the sack after each draw.)
9. Ask the class how many times this procedure needs to be repeated to draw each color once.
10. Discuss similarities between this activity and buying pretzels to get all the different stickers.
11. In groups, students will repeat this simulation as many times as possible in the time given and record results.
12. As a class, create a frequency table to show the frequencies of numbers of tries it took to get one cube of each color.

Exploration:

- Are you drawing any one color more than others?
- When is the experiment over?
- What is the fewest number of draws you've had to make so far?
- What is the fewest number of draws possible?
- What is the most number of draws you've had to make so far?
- What is the most number of draws possible?
- Have any of your group's experiments given you the same results?
- Why is it necessary to replace the cube drawn after each draw?

Extensions:

- With 10 cubes of each of the six colors, repeat the experiment several times. Compare results to original experiment.
- Role play the purchase of pretzels with real sacks and stickers. (This could be put in a learning center and done over an extended period of time.)
- Read *Charlie and the Chocolate Factory* for language arts extension.

Summary:

- How can you explain the relationship between the experiments with the cubes and the pretzel problem?
- How can you justify the validity of the decisions you reached about the pretzel situation from the results of these experiments?
- What results from the experiments would you expect if the numbers of cubes differed; if there were 3, 4, or 10 colors, for example, instead of 6?
- Suppose you used a die instead of six cubes. Would this have been a sensible simulation? Why or why not?
- What do you think you have learned from this lesson?

Assessment

Questions:

- How did you organize yourself as a group?
- How did you organize your data? Would there be a better way to organize your data?
- How did your group work together? How could you improve?
- (See summary questions.)

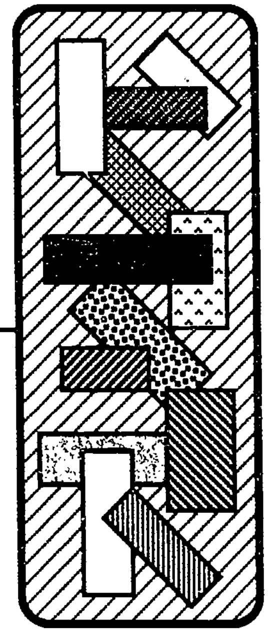
Observations:

- Were students recording their draws accurately?
- Were students making predictions as they worked?
- Did students stop drawing after they received one cube of each color and begin a new experiment?
- Did students use appropriate vocabulary?

Tasks:

- Write in your mathematics journal how many boxes of pretzels you think you would have to purchase o get a complete set of stickers, using what you have learned from the paper sack and cube experiment.
- Write and compare the pretzel situation to the paper sack experiment.

Evaluation



Philosophy

NCTM's *Professional Standards for Teaching Mathematics* and *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) emphasize the connection between assessment of students and analysis of instruction. In other words, mathematics teachers should monitor students' learning (both formatively and summatively) in order to assess and adjust teaching. Teachers must observe and listen in order to tailor teaching strategies. Information about what students are understanding should be used to revise and adapt short- and long-range plans, and students' understandings should guide teachers in shaping the learning environment. Also, teachers are responsible for describing students' learning to administrators, parents, and students themselves.

Students' mathematical power depends on various understandings, skills, and dispositions. The development of students' abilities to reason mathematically—to conjecture, justify, and revise based on evidence and to analyze and solve problems—must be assessed. A student's disposition toward mathematics (confidence, interest, perseverance, etc.) is also a key dimension that teachers should monitor.

The importance of using assessment to improve instruction is crucial. Information should be gathered from multiple sources using numerous assessment techniques and modes that are aligned with the curriculum. Assessment techniques must reflect the diversity of instructional methods implied in the *Standards* and the various ways students learn and process information. Instructional decisions should be based on this convergence of information from different sources.

In summary, the following aspects of students assessment and program evaluation should receive increased and decreased attention (NCTM, 1989):

Increased Attention	Decreased Attention
• Assessing what students know and how they think about mathematics	• Assessing what students do not know
• Having assessment be an integral part of teaching	• Having assessment be simply counting correct answers on tests for the sole purpose of assigning grades
• Focusing on a broad range of mathematical tasks and taking a holistic view of mathematics	• Focusing on a large number of specific and isolated skills organized by a content-behavior matrix
• Developing problem situations that require the applications of a number of mathematical ideas	• Using exercises or word problems requiring only one or two skills
• Using multiple assessment techniques, including written, oral, and demonstration formats	• Using only written tests
• Using calculators, computers, and manipulatives in assessment	• Excluding calculators, computers, and manipulatives from the assessment process
• Evaluating the program by systematically collecting information on outcomes, curriculum, and instruction	• Evaluating the program only on the basis of test scores
• Using standardized achievement tests as only one of many indicators of program outcomes	• Using standardized achievement tests as the only indicator of program outcomes

Types of Evaluation

While paper and pencil tests are one useful medium for judging aspects of students' mathematical knowledge, teachers need information gathered in a variety of ways and using a range of sources. Observing, interviewing, and closely watching and listening to students are all important means of assessment. While monitoring students, teachers can evaluate the learning environment, tasks, and discourse that have been taking place. Using a variety of strategies, teachers should assess students' capacities and inclinations to analyze situations, frame and solve problems, and make sense of concepts and procedures. Such information should be used to assess how students are doing, as well as how well the tasks, discourse, and environment are fostering students' mathematical power and then to adapt instruction in response.

Assessment instruments and techniques should be properly aligned with the curriculum to enable educators to draw conclusions about instructional needs, progress in achieving the goals of the curriculum, and the effectiveness of a mathematics program. That is, the content, processes, and skills assessed must reflect the goals, objectives, and breadth of topics specified in the curriculum. The particular emphases of the assessment should reflect the emphases of instruction. For example, primary children, whose understanding of fractions is closely tied to the use of physical materials, should be encouraged to use such materials to demonstrate their conceptual knowledge. Assessment items need to be structured around the central ideas of the curriculum and need to provide opportunities for students to demonstrate their understanding of the connections among major concepts. In addition, assessment must reflect the relative emphasis placed on technology during instruction; to the extent that calculators and computers have been important during instruction, they should also be available during assessment.

Assessment techniques suggested in the *Standards* include multiple-choice, short-answer, discussion, and open-ended questions; interviews; homework; projects; journals; essays; portfolios; presentations; and dramatizations. Assessment can occur during and after whole-group explorations, during whole-group discussions, in sharing sessions, during individual conferences, during small-group conferences, while students are working on projects, after completion of projects, when students are engaged in self-evaluation tasks, and continually while students are explaining, justifying, debating, and questioning ideas and concepts.

Using Portfolios. Student portfolios are becoming more prevalent as a means of keeping a record of student progress in mathematics. Teachers have always kept folders of students' work, but portfolios should have more focus and be more important for assessment. An assessment portfolio is a planned selection of a student's work collected throughout the school year. Teachers as well as students should be allowed to choose the items to be included in portfolios, since it gives a good indication of what is valued in the work the students do throughout the school year. A portfolio might include samples of student-produced written descriptions of the results of practical or mathematical investigations; pictures and dictated reports from younger students; extended analyses of problem situations and investigations; descriptions and diagrams of problem-solving processes; statistical and graphic representations; responses to open-ended questions or homework problems; group reports and photographs of student projects; copies of awards or prizes; video, audio, and computer-generated examples of student work; and other material based on project ideas developed with colleagues.

Using Writing. Communication in mathematics has become important as we move into an era of a thinking curriculum. Journals, logs, problem-solving notebooks, explanations, justifications, and reflections are ways to include writing in the mathematics curriculum. Students should be urged to discuss ideas with each other, and to ask questions, to diagram and graph problem situations for clarity. Writing in mathematics classes, once rare, is now vital. In particular, mathematics journals can include the following:

- vocabulary definitions written in the student's own words along with explanations of how the terms are used in mathematics

- rules or procedures written as if explained to a friend in a letter or to another student who was absent during the instruction

- free writing, including what students think they will learn in an experience, descriptions of accomplishments, how students can use what they've learned, what isn't fully understood or is causing difficulty, examples in the real world related to the mathematics learned, a discovery made or additional ideas and conjectures related to the topic, and what else students might want to learn about.

These writing experiences are also important mathematics learning experiences in that they:

- help students become more active in their own learning

- help students internalize what they are learning to make it more meaningful

- allow students to express their feelings and attitudes toward mathematics

- give students a source they can use for studying

- allow students to reflect upon and clarify their own thinking

- give students the opportunity to share with each other what they are learning, also allowing them to learn from one another

- allow students to go beyond what they are learning in class and to make conjectures and connections

- give students the opportunity to think of mathematics as existing outside the classroom

- give students the opportunity to communicate with the teacher in an informal setting

- give the teacher an idea of how students are thinking

- allow the teacher to informally assess student learning (whether it be pre- or post-assessment)

Using Teacher Observations. Teacher observations can be broken down into two levels: formal and informal. Formal observations include checklists, comment cards, and summaries. Informal observations include mental notes. Students should be observed both individually and as they work in groups. When using observations, a teacher should look for students' learning styles, students' ideas, communication techniques, cooperation strategies, and use of manipulatives. Some possible questions that can guide observations of students doing mathematics are:

- Does the student consistently work alone or with others?

- Does the student try to explain organizational and mathematical ideas?

- Does the student synthesize and summarize his or her own or a group's thinking?

- Does the student work with the group to agree on a plan or structure for tackling the task?

- Individually or within the group, does the student choose and use appropriate manipulatives?

Using Questioning. Asking the right question is an art to be cultivated by all educators. Low-level quizzes that ask for recall or simple computations are over used and over done. Using good, high-level open-ended questions that give students a chance to think are one of the goals of mathematics assessment. These questions might be used as teaching or leading questions as well as for assessment purposes. Both questions and responses may be oral, written, or demonstrated by actions taken. When using oral questions, the teacher can prepare a list of possible questions ahead of time. (For examples, see the sample activities in the previous section.) The teacher should allow for plenty of wait time. The teacher may keep a written record of observations during the questioning time to use for formal assessment. Questioning for assessment should occur in several places during instruction:

- during introductory activities to assess students' prior knowledge and experience
- during exploration to focus students' attention on important concepts and connections
- after instruction, in order for students to summarize results, reflect on their experience, and clarify their thoughts

Using Student Presentations. Student presentations can take many forms, including oral explanations, oral presentations, and projects. One of the best ways to assure the connection between instruction and assessment is to embed assessment into instruction. When students become involved in projects or investigations, assessment becomes natural and invisible. Student presentations may be related to connections within mathematics and connections outside mathematics. When evaluating student presentations, the teacher should look for whether the student can identify and define the problem; make a plan; collect needed information; organize the information and look for patterns; discuss, review, revise, and explain results; and produce a quality product or report.

Using Performance Assessment. Performance assessment involves giving a group of students, or an individual student, a mathematical task that may take from half an hour to several days to complete or solve. The object of this form of assessment is to look at how students are working, as well as at the completed tasks or products. Performance assessment requires the teacher to look at how students solve a problem. Performance activities may be videotaped, tape recorded, or recorded in writing. The task might be from any mathematical content area and might include some connections such as with science, social studies, language arts, or fine arts. Performance assessment is an excellent place for students to use manipulatives to demonstrate understanding of mathematics content. Information from performance assessment can be recorded using rubrics that assign point values to important aspects of the problem-solving process. For example, the following assessment criteria could be used during observation or based on written work to judge a student's involvement in problem solving:

1. Understanding the Problem:

0 points	Does not understand the problem
1 point	Misunderstands part of the problem
2 points	Completely understands the problem

2. Choosing and Implementing a Solution Strategy

0 points	Makes no attempt or uses a totally inappropriate strategy
1 point	Chooses a partly correct strategy based on interpreting part of the problem incorrectly
2 points	Chooses a strategy that could lead to a correct solution if used without error

3. Getting the Answer

0 points	Gets no answer or a wrong answer based on an inappropriate solution strategy
1 point	Makes a copying error or computational error; gets partial answer for a problem with multiple answers; or labels answer incorrectly
2 points	Gets correct solution

Some excellent resources on assessment, in addition to the NCTM *Curriculum and Evaluation Standards*, include *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (1991) and *Assessment alternatives in Mathematics* (Stenmark, 1989).

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Geddes, D. (1992). *Geometry in the middle grades: Addenda Series, Grades 5-8*. Reston, VA: National Council of Teachers of Mathematics.

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Parker, T. (1984). *In one day.* Boston, MA: Houghton Mifflin.

Pollard, J. (1985). *Building toothpick bridges.* Palo Alto, CA: Dale Seymour Publications.

Popping with power. (1987). Fresno, CA: AIMS Education Foundation.

Stenmark, J. K. (1989). *Assessment alternatives in mathematics.* Berkeley, CA: Lawrence Hall of Science, University of California.

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Wahl, J. and Wahl, S. (1976). *I can count the petals of a flower.* Reston, VA: National Council of Teachers of Mathematics.

Children's Trade Books

Numerous children's books have the potential for motivating rich mathematics activities. This short list gives the bibliographic information of the books mentioned in the activities in this series of curriculum guides for elementary mathematics (Grades 1 - 5).

Anno, M. (1982). *Anno's counting house*. New York: Philomel Books.

Anno, M. (1983). *Anno's mysterious multiplying jar*. New York: Philomel Books.

Aardema, V. (1976). *Why mosquitoes buzz in people's ears*. New York: Dial Books for Young Readers.

Bemelmans, L. (1960). *Madeline*. New York: Viking Children's Books.

Bishop, C. H. (1938). *Five Chinese brothers*. New York: Coward.

Brier, C. (1983). *The shoemaker and the elves*. New York: Lothrop, Lee & Shepard.

Carle, E. (1977). *The grouchy ladybug*. New York: Scholastic, Inc.

Carle, E. (1989). *The very hungry caterpillar*. New York: Putman.

Dahl, R. (1964). *Charlie and the chocolate factory*. New York: Alfred Knopf.

de Paola, T. (1978). *The popcorn book*. New York: Holiday House.

Ehlert, L. (1990). *Fish eyes: A book you can count on*. San Diego: Harcourt, Brace, Jovanovich.

Faucher, E. (1989). *Honey, I shrunk the kids*. New York: Scholastic, Inc.

Flournoy, V. (1985). *The patchwork quilt*. New York: Dial Books for Young Readers.

Freeman, D. (1968). *Corduroy*. New York: Viking Press.

Giganti, Jr., P. (1992). *Each orange had 8 slices*. New York: The Trumpet Club.

Haley, G. E. (1971). *Noah's ark*. New York: Atheneum.

Hogrigan, N. (1966). *Always room for one more*. New York: Holt.

Hulme, J. N. (1991). *Sea squares*. New York: Hyperion Books.

Hutchins, P. (1987). *The doorbell rang*. New York: Scholastic, Inc.

Lobel, A. (1983). *Frog and toad are friends*. New York: Harper.

Mathews, L. (1978). *Bunches and bunches of bunnies*. New York: Scholastic, Inc.

McMillan, B. (1991). *Eating fractions*. New York: Scholastic, Inc.

Owen, A. (1988). *Annie's one to ten.* New York: Alfred Knopf.

Parish, P. (1963). *Amelia Bedelia.* New York: Harper and Row.

Peterson, J. (1967). *The Littles.* New York: Scholastic, Inc.

Potts, J. (1992). *The house that makes shapes.* New York: Hyperion.

Prelutsky, J. (1984). *New kid on the block.* New York: Greenwillow.

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Price, L. (1990). *Aida.* New York: Harcourt, Brace, Javonovich.

Sendak, M. (1962). *Chicken soup with rice.* New York: Harper and Row.

Silverstein, S. (1981). *A light in the attic.* New York: Harper and Row.

Silverstein, S. (1974). *Where the sidewalk ends.* New York: Harper and Row.

Singer, M. (1991). *Nine o'clock lullaby.* New York: HarperTrophy.

Steptoe, J. (1980). *Daddy is a monster . . . sometimes.* New York: HarperTrophy.

Ward, C. (1988). *Cookie's week.* New York: Putnam.

West, C. (1987). *Ten little crocodiles.* New York: Barron's.

Yolen, J. (1987). *Owl moon.* New York: Philomel Books.

Software

The following list contains bibliographic information for the software packages mentioned in this series of curriculum guides for elementary mathematics (Grades 1 - 5). Other appropriate software may be obtained from these and other companies.

Blockers and Finders from WINGS for learning/Sunburst Communications, 1600 Green Hills Road, P.O. Box 660002, Scotts Valley, CA 95067-0002.

Geometric preSupposer from WINGS for learning/Sunburst Communications, 1600 Green Hills Road, P.O. Box 660002, Scotts Valley, CA 95067-0002.

Hands-On Math: Volumes 1, 2, and 3 from Ventura Educational Systems, 3440 Brokenhill Street, Newbury Park, CA 91320.

Suggested Manipulatives

The following is a list of the manipulative materials used in the activities in this series of curriculum guides for elementary mathematics (Grades 1 - 5):

Calculators
Base ten blocks
Coins and bills (play or real money)
Interlocking cubes
Colored tiles
Pattern blocks
Cuisenaire rods

Graphing floor mat
Polyhedral dice (including the regular cube)
Colored chips
Two-color counters

Attribute blocks
Geoblocks
Geoboards
Tangrams
Plastic mirrors
Wooden or plastic models of geometric solids

Balance scales and masses (customary and metric)
Spring scales
Tape measures (customary and metric)
Rulers (customary and metric)
Meter sticks and yardsticks
Trundle wheels
Graduated cylinders
Measuring cups and spoons
Stopwatches

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TITLE VI, CIVIL RIGHTS ACT OF 1964; THE MODIFIED COURT ORDER, CIVIL ACTION 5281, FEDERAL DISTRICT COURT, EASTERN DISTRICT OF TEXAS, TYLER DIVISION

Reviews of local education agencies pertaining to compliance with Title VI Civil Rights Act of 1964 and with specific requirements of the Modified Court Order, Civil Action No. 5281, Federal District Court, Eastern District of Texas, Tyler Division are conducted periodically by staff representatives of the Texas Education Agency. These reviews cover at least the following policies and practices:

(1) acceptance policies on student transfers from other school districts;

(2) operation of school bus routes or runs on a nonsegregated basis;

(3) nondiscrimination in extracurricular activities and the use of school facilities;

(4) nondiscriminatory practices in the hiring, assigning, promoting, paying, demoting, reassigning, or dismissing of faculty and staff members who work with children;

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